

# Dual Sourcing with Price Discovery<sup>\*</sup>

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## Abstract

We consider a (standard) reverse auction for dual sourcing and propose to determine both the providers' shares and the reserve price endogenously, depending on the suppliers' bids. Our benchmark considers a two-stage game of complete information. After a first round of bidding, the two most competitive suppliers advance to the second stage and compete again with a refined reserve price, which is based on the lowest price of the excluded providers. We show that at the first stage providers reveal their costs truthfully. At the second stage suppliers balance a trade-off between increasing their share and raising their mark up. Surprisingly, when discarded suppliers are competitive enough, the procedure not only allows taking advantage of dual sourcing but also generates lower procurement expenditures than a standard auction for sole sourcing. We also consider extensions of the benchmark model, including to situations in which providers have private information about their costs.

Keywords: Dual Sourcing, Procurement Auctions, Contests, Price Discovery

Journal of Economic Literature Classification Numbers: **D44**, **D47**.

# 1 Introduction

Procurement is an important part of economic activity.<sup>1</sup> It is uncontroversial that procurement should minimize costs and from a theoretical point of view the optimal mechanism is well understood. This mechanism, however, requires information about the bidders' value distributions in order to determine the optimal reserve prices. In addition, it is not anonymous, as it does not treat bidders in the same way. Consequently, in practice relatively simple traditional reverse auctions are used.<sup>2</sup> In this paper we propose a new procurement procedure that is anonymous, builds on reverse auction formats used in real-life procurement markets, and has the potential to reconcile the conflicting aims of expenditure minimization and dual sourcing.

In some procurement markets it is not only important to minimize procurement costs but also to avoid dependence on a single provider. Having only one supplier risks that the buyer is 'locked in' with one provider and experiences shortage in the case that this supplier cannot fulfil his obligations. Currently, for example, the state of Texas buys influenza vaccines from Novartis and Sanofi Pasteur, while meningococcal vaccines are provided by Sanofi Pasteur and Glaxosmithkline.<sup>3</sup> Similarly, in the private sector, Nokia and Toyota follow a dual sourcing strategy in order to reduce supply chain risk.<sup>4</sup> Since having several providers might require to forgo economies of scale or to buy from providers with different efficiency levels, conventional wisdom holds that there is a trade-off between expenditure minimization and a dual (or even multiple) sourcing strategy.<sup>5</sup> The main contribution of this paper is to propose a procurement procedure

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<sup>1</sup>Procurement of government contracts represents 19.96% of GDP for OECD countries and 14.48% for non-OECD countries, while the value of procurement transactions in the private sector is estimated to be even larger than in the public sector, see OECD (2002) and Dimitri et al. (2006).

<sup>2</sup>In fact, Milgrom (2004) summarizes the so-called "Wilson doctrine" as arguing "that useful auction designs must be independent of the fine details of unknowable bidder valuations and beliefs" (p. 165). As Krishna (2010) notes, "Any mechanism that depends on the fine details of buyers' distributions would be difficult to implement in practice" (p. 75). Krishna defines auctions in his book to be the subset of mechanisms that are both anonymous and universal, where a universal mechanism is one that may be used to sell any good, as its rules do not depend on any specific details, like the value distributions. Carpineti et al. (2006), Tunca and Wu (2009), or Krasnokutskaya (2011) report the use of simple traditional reverse auctions in procurement.

<sup>3</sup>See <http://www.txsmartbuy.com/contracts/view/1741> and <https://comptroller.texas.gov/purchasing/>, both accessed on 05/08/2018. Dual sourcing is also used in defense procurement in the US and by Consip, the Italian central procurement agency, see Grimm et al. (2006) and Albano et al. (2006a). Carpineti et al. (2006) report that both the US Federal Acquisition Regulation and the relevant EU Directive regulate the concept of multiple awarding.

<sup>4</sup>See <http://www.scdigest.com/assets/newsViews/07-08-15-1.php?cid=1178> and <http://www.scdigest.com/ontarget/12-03-07-2.php?cid=5576>, both accessed on 05/08/2018. For the example of Toyota, see also Richardson (1993) and Albano et al. (2006a). Albano et al. (2006a) also report that Soletron, an electronics manufacturer, uses dual sourcing.

<sup>5</sup>See Albano et al. (2006a) or Engel et al. (2006). Albano et al. (2006a) write, "However, dual sourcing

that results in dual sourcing but has the potential to avoid this trade-off.<sup>6</sup>

A commonly employed dual sourcing strategy is as follows.<sup>7</sup> The buyer announces (i) which objects or services the supply contract contains; (ii) the reserve price (or budget constraint, or bid ceiling); and (iii) an ex-ante specified proportion, say equal split, in which the two winning providers share the supply contract. Providers make their bids and the two suppliers proposing the lowest prices are chosen. This procedure results in the Vickrey outcome in which the two lowest-cost providers win a share at the price offered by the third lowest cost provider.<sup>8</sup> In what follows we will refer to this sourcing strategy as Vickrey auction for dual sourcing.

We propose a procurement procedure that builds on this format but determines both the providers' shares and the reserve price endogenously, depending on the suppliers' bids. The procedure assigns positive shares to the two most competitive bidders.<sup>9</sup> Discarded prices, however, are used for price discovery. The lowest discarded price is used to replace the initial reserve price (or budget). As a consequence the initial reserve price does not affect the equilibrium outcome and the buyer can save costly resources when determining it. In addition, the procedure avoids setting the initial reserve price too low and accidentally deterring participation of suppliers in the auction. To describe the assignment rule for shares more precisely, define a supplier's bid to be the difference between the refined reserve price and the price of this supplier. Shares are assigned depending on the relative difference of the bids of suppliers, as a percentage of the largest bid (submitted by the supplier proposing the lowest price).

We start by considering a benchmark and formalise this idea as a two-stage game of complete information.<sup>10</sup> In the first stage, the price discovery stage, providers propose prices. Based on these prices two suppliers are chosen to compete in the second stage

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does not come without costs. In particular, splitting procurement (i) reduces the economics of scale of each supplier, ... and (ii) ... almost certainly includes one supplier that is not at least cost" (p. 110). Engel et al. (2006) state, "The disadvantage [of multiple sourcing] is that the price is in general higher than with single sourcing" (p. 330).

<sup>6</sup>A related trade-off between expenditure minimization and minority representation appears in procurement when there are affirmative action considerations and our procedure might be of interest in this context, see Alcalde and Dahm (2013).

<sup>7</sup>See Engel et al. (2006), Tunca and Wu (2009) or the clearing-price auction in Cramton et al. (2015).

<sup>8</sup>See Engel et al. (2006), Tunca and Wu (2009) and Wambach (2002).

<sup>9</sup>Notice that even though only two suppliers win shares, any provider who wishes to participate can do so. In this sense there is free entry and equal treatment of providers, which is normatively appealing.

<sup>10</sup>As observed by Albano et al. (2006b), Carpineti et al. (2006) and Tunca and Wu (2009), different forms of two-stage processes are frequently used in real life procurement situations, including in the privatization of the formerly state-owned industrial conglomerate ENI. One form has a second round of bidding among the winners of the first stage. The complete information setting is considered appropriate when providers know each other well (Moldovanu and Sela, 2003). Examples are construction contracting (Bernheim and Whinston, 1986) or when technology can be considered to be stable (Anton and Yao, 1992, p. 691).

with the refined reserve price. In this contest stage the two providers can adjust their initial price proposals provided that the revised price is below the refined reserve price. We show that there is a unique weakly-stage dominant equilibrium.<sup>11</sup> Interestingly, providers reveal their costs truthfully at this price discovery stage. This guarantees that procurement fulfils a (minimal) efficiency property, in the sense that the two lowest cost providers are assigned procurement shares, rather than that a supplier with higher cost obtains a part of the supply contract.

Moreover, we show that our procedure has the potential to avoid the trade-off between expenditure minimization and a dual sourcing. To formalise this we compare the procurement costs of our procedure to a standard Vickrey auction for sole sourcing, in which procurement costs are equal to the second lowest cost.<sup>12</sup> In other words, we ask when a buyer, who does not necessarily value dual sourcing by itself, prefers to use our procedure. She will do so when procurement costs are lower with our procedure than with sole sourcing, in which case the trade-off between expenditure minimization and dual sourcing disappears. We show that our procedure has the potential to generate very competitive procurement, because the winning suppliers are not only concerned with outbidding their rivals and obtaining a positive share, but also compete to increase the relative size of their shares. More precisely, we establish that if discarded suppliers are competitive enough (as measured by the cost difference between the second and the third lowest cost providers), then we can guarantee that procurement costs are lower than in a standard Vickrey auction for sole sourcing.

The benchmark model can be extended in different ways. Since there are procurement settings that the assumption of complete information does not seem to fit well, we offer three extensions that introduce incomplete information into our framework. In all three extensions, we show that the existence of an equilibrium with truthful reserve price discovery continues to hold. The first two extensions consider the two-stage process of the benchmark model and either provide conditions on the informational environment or assume that suppliers are averse to loser regret. The third extension dispenses both with restrictions on the informational environment and the assumption of loser regret but varies the auction procedure employed. More precisely, in the third extension we follow Edelman et al. (2007) and Alcalde and Dahm (2013) and con-

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<sup>11</sup>This is an equilibrium in which, at the first stage, each supplier reports a price that weakly dominates any other report.

<sup>12</sup>Carpinetti et al. (2006) report that procurement auctions are usually organized as first-price auctions and that many organisations do not determine the reserve price in the way the optimal mechanism would require. A similar benchmark is used in Ewerhart and Fieseler (2003) and Alcalde and Dahm (2013). Notice that it is sufficient to compare to a standard auction to show that it can improve over the aforementioned Vickrey auction for dual sourcing (with exogenous shares). This is so, since the Vickrey auction for dual sourcing generates higher procurement costs than the standard Vickrey auction for sole sourcing, see Engel et al. (2006), Tunca and Wu (2009) and Wambach (2002).

sider a variant of a reverse English (or Japanese) auction with rules that resemble the two-stage process of the benchmark model.<sup>13</sup> In such an auction the buyer decreases the price continuously over time. Although providers initially do not have information about each other, during the course of the auction all the relevant information is revealed so that in equilibrium each supplier obtains the same share and payoffs as in the benchmark. Interestingly, the dominance properties of the equilibrium in the benchmark translate to the dynamic setting, and as a result our equilibrium differs from Edelman et al. (2007) in that it is in weakly dominant strategies. This implies that the procurement procedure can be used in different scenarios concerning what providers know about each others' costs, provided the assumption of private values holds. In the last extension, we argue that when the buyer values dual sourcing by itself, our procedure becomes more attractive. We also discuss the risk in setting the reserve price optimally, based on prior information, rather than refining it through price discovery. An Appendix considers a natural extension to multiple sourcing.

Our paper determines the providers' shares and the reserve price of the Vickrey auction for dual sourcing endogenously. In doing so we build on Alcalde and Dahm (2013) who restricted to the case of two suppliers and endogenized the providers' shares only. The assignment rule in the present paper reduces for two suppliers to a special case (more precisely the so-called case of unit elasticity) of the *Contested Procurement Auction* (CPA henceforth) that we proposed in our earlier paper.<sup>14</sup> In the present paper we show that in a model with more providers, the buyer can take advantage of price discovery in order to determine the reserve price. This reduces the information required in practice and avoids deterring participation by setting the reserve price too low. In addition, the existence of a competitive pool of potential suppliers benefits the buyer, because the trade-off between expenditure minimization and a dual sourcing disappears in less demanding circumstances than in our earlier paper.

The assignment of shares also connects our paper to the literature on share and split-award auctions that explores conditions under which sole sourcing is more advantageous than a split-award (Wilson, 1979; Bernheim and Whinston, 1986; Anton and Yao, 1989, 1992; Perry and Sákovics, 2003). A major difference is that our allocation

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<sup>13</sup>This auction is a reverse auction variant of the English auction analysed in Milgrom and Weber (1982). It differs from other descending clock auctions for divisible goods in that bidders do not specify quantities. In our auction quantities follow from the relative prices, which are determined by the relative drop out decisions. As observed by Albano et al. (2006b) and Tunca and Wu (2009) English reverse auctions have become more popular in real life procurement situations. The former authors offer practical guidelines for descending clock auctions.

<sup>14</sup>The name comes from a related solution for bankruptcy situations, the so-called *Contested Garment Principle*, see Dagan (1996). Alcalde and Dahm (2013) consider a family of assignment rules that differ in the sensitivity of a supplier's procurement share with regard to his price. From a normative point of view, the CPA can be motivated through a connection to the framework of bargaining with claims, because it coincides with the relative claim-egalitarian solution (see Corchón and Dahm, 2010).

rule for procurement shares imposes a particular structure on the trade-off a supplier faces when deciding on his price. This allows to induce strong competition and to improve upon sole sourcing.<sup>15</sup> In contrast, Anton and Yao (1989, 1992) emphasize the existence of high price split-award equilibria.<sup>16</sup>

The endogenous assignment of shares with our procedure also relates to the determinants of the size of lots in combinatorial auctions for procurement of multiple lots, see Grimm et al. (2006) and the literature therein. Similar to our procedure, splitting the supply contract in several lots can reduce the risk of lock-in by inducing dual sourcing. An important difference is that the size (and number of) lots are determined in advance, before bidding takes place. Consequently, suppliers are only concerned with outbidding rivals but not with increasing their share, as with our procedure. Instead, the optimal size of lots resolves a trade-off between facilitating entry through smaller lots and taking advantage of economics of scale with larger lots.<sup>17</sup>

In our benchmark providers reveal their costs truthfully at the price discovery stage. This is because, at the contest stage, providers are constrained by the refined reserve price (which depends on the lowest discarded price), rather than by their (own) first stage price. Consequently, our two-stage auction results in efficient entry to the second stage. This relates our procedure to the models of two-stage auctions of indicative bidding by Ye (2007) and Quint and Hendricks (2017). These papers show in a different model that efficient entry to the second stage depends on whether first-stage bids are binding or not. The literature has also explored other ways to constrain the second stage participants. In Perry et al. (2000) providers are constrained by their first stage prices and the auction yields the same expected revenue as the open ascending (English) auction. In Fujishima et al. (1999) suppliers are constrained by the least competitive

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<sup>15</sup>Our paper also relates to the literature on optimal design of procurement auctions (Myerson, 1981; Dasgupta and Spulber, 1990; Maskin and Riley, 2000). As mentioned before, however, we are interested in the practical applicability of our procurement mechanism, which in general leads to the use of traditional reverse auctions. Consequently, our procurement procedure builds on the Vickrey auction for dual sourcing (with exogenous shares).

<sup>16</sup>As described by McMillan (2003, p. 140) these equilibria fit well procurement by the Pentagon during the 1980's:

Unfortunately, the “competition” under dual sourcing was counterproductive. Each firm’s best strategy often was to bid higher than its rival, for a firm could earn more by getting the smaller share at a high price than by being the low bidder (Anton and Yao, 1992). Theorists pointed out the flaw in dual sourcing at a 1986 Rand Corporation conference on defense procurement. In 1989, the Pentagon’s Inspector General concluded that dual sourcing had failed because it was “conducive to price gaming.”

<sup>17</sup>Inspired by Japanese auto makers Toyota and Nissan, Richardson (1993) and Richardson and Roumasset (1995) discuss the use of dual sourcing as an instrument to improve performance of providers. This provides incentives for high performance when some part of the share of a low performing provider is reallocated to a high performing provider. In our model the quality of provision plays no role.

excluded bid (rather than the most competitive as in our procedure) and the auction is Nash outcome equivalent to the Japanese auction.<sup>18</sup>

This paper is organized as follows. The next section introduces the procurement problem and the rule used to assign shares. Section 3 completes this model as a two-stage process of complete information and analyses the equilibrium. Section 4 considers extensions of this benchmark, including to incomplete information. Lastly, Section 5 offers some concluding remarks. All the proofs are relegated to the Appendix, which also includes a discussion of multiple-sourcing.

## 2 The Procurement Problem

A buyer wishes to buy a certain quantity of a perfectly divisible good. The size of the supply contract is normalized to 1. The buyer has an exogenously determined budget  $b$ . This budget specifies the maximum amount she can spend and will be interpreted as the reserve price. There are  $n > 2$  potential suppliers. Each provider has a constant average cost  $c_i \geq 0$  and sufficient capacity to produce the whole supply contract.

As explained in the Introduction, we make the benchmark assumption that the buyer's only objective is to minimize procurement costs and compare our procedure to a standard (Vickery) second-price auction for sole sourcing, in which procurement costs are equal to the second lowest cost. We discuss the buyer's objective function further in Subsection 4.4.

We propose next an extension of the CPA in Alcalde and Dahm (2013) to  $n$  providers. Suppose that, given the suppliers' costs  $C = (c_1, \dots, c_i, \dots, c_n)$  and the reserve price  $b$ , suppliers choose prices  $P = (p_1, \dots, p_i, \dots, p_n)$  at which they are willing to provide the good. In order to introduce the *Generalized Contested Procurement Auction* (GCPA), assume also that prices are increasingly ordered, i.e. for each  $i < n - 1$ ,  $p_i \leq p_{i+1}$ .<sup>19</sup> The GCPA allocates procurement shares to providers in the following way:

- (a) if  $p_1 < b$ , then each supplier receives

$$\varphi_i^{GCPA}(P|b) = \sum_{j=i}^n \frac{\min\{p_{j+1}, b\} - \min\{p_j, b\}}{j(b - p_1)}, \quad (1)$$

with  $p_{n+1} = b$ ;

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<sup>18</sup>Our analysis of the two-stage procedure also relates to elimination tournaments in which only winners proceed to later stages (Fullerton and McAfee, 1999; Fu and Lu, 2012), and the buyer's decision between dual and sole sourcing relates to the choice between a lottery contests and an all-pay auction (Fang, 2002; Epstein et al., 2013; Franke et al., 2014; Matros and Possajennikov, 2016).

<sup>19</sup>If necessary relabel the set of providers.



(b) if  $p_1 = b$ , then all suppliers reporting the lowest price share the total amount equally at that price; and

(c) if  $p_1 > b$ , then there is no provision and each provider's share is zero.

Notice that the recursive allocation rule in (1) does not require any information about the good or suppliers' costs and is independent of providers' labels. It is hence universal and anonymous, fulfilling the criteria for practical applicability in Krishna (2010). It also assigns equal shares in case of ties and implicitly assumes a feasibility condition, because if a provider's price is higher than the buyer's budget constraint, his share is zero.

This assignment rule in (1) reduces to the CPA for  $n = 2$ .<sup>20</sup> Alcalde and Dahm (2013) have shown that when two providers choose prices simultaneously, the CPA has the potential to generate low procurement costs. More precisely, when

$$\frac{c_2 - c_1}{b - c_2} > \left( \frac{13}{8} + \frac{5}{8}\sqrt{17} \right) \approx 4.20, \quad (2)$$

then in the unique equilibrium procurement costs are lower than  $c_2$ , the buyer's expenditures in a Vickrey auction for sole sourcing. Notice that condition (2) is the more likely to be fulfilled, the greater the heterogeneity of the firms, relative to how competitive the reserve price is, as measured by the difference  $b - c_2$ . This implies that the buyer might benefit from decreasing  $b$ .

This is the starting point for the present paper. We use the third lowest price to endogenize the reserve price and to reduce  $b$ .<sup>21</sup> For completeness we define the third lowest price formally as follows.

**Definition 1** For any  $P = (p_1, \dots, p_i, \dots, p_n)$ , with  $p_i \in [0, b]$  for each supplier  $i$ , the third lowest price  $\hat{t}(P) \in \{p_1, \dots, p_i, \dots, p_n\}$  is such that

- (a)  $\{i: p_i \leq \hat{t}(P)\}$  has at least three suppliers; and
- (b)  $\{h: p_h < \hat{t}(P)\}$  has at most two suppliers.

To illustrate this definition, notice that when  $p_1 \leq p_2 \leq p_3 \leq \dots \leq p_n \leq b$  holds, we have that  $\hat{t}(P) = p_3$  even if  $p_2 = p_3$  or  $p_1 = p_3$ . We now use the so defined third lowest price to replace  $b$  in the GCPA, that is, the allocation rule is  $\varphi_i^{GCPA}(P|\hat{t}(P))$ . Provided  $p_2 < p_3$ , this results in dual sourcing, as the lowest and the second lowest bidder receive positive shares. We discuss a natural extension to multiple sourcing in Appendix A.5.

<sup>20</sup>In addition, it preserves the desirable mathematical properties of the rule for two suppliers. Specifically, it is continuous (everywhere but when all the providers set their price equal to the reserve price  $b$ ) and the shares are monotonic in bids. Lastly, the assignment is homogeneous, that is, it is independent of the numéraire employed.

<sup>21</sup>Decreasing  $b$  to  $b'$  based on prior information risks reducing the reserve price too much, deterring entry and inhibiting competition, see Carpineti et al. (2006). Using the third lowest price to reduce  $b$  avoids this. We discuss this issue further in Subsection 4.4.

### 3 Truthful Reserve Price Discovery

As observed in the Introduction, some commonly employed procurement procedures include two-stage processes that have a second round of bidding among the winners of the first stage. In this section we propose such a mechanism and complete the model of the previous section as a two-stage process of complete information.<sup>22</sup> More precisely, we assume that it is common knowledge among suppliers that

$$c_1 < c_2 < \dots < c_n < b. \quad (3)$$

Similar to Bernheim and Whinston (1986) or Anton and Yao (1989), we assume that the buyer does not know the costs of providers.<sup>23</sup> In the next section we discuss extensions of this basic framework.

The two-stage game works as follows. In the first stage, the price discovery stage, suppliers are asked to reveal their true marginal cost. The two most efficient providers are selected to compete in the second stage. In this second stage, the contest stage, the two providers compete in the *Contested Procurement Auction* CPA (Alcalde and Dahm, 2013), with the buyer's reserve price endogenously determined by the third lowest report in the price discovery stage.

We show that this procedure induces truthful reporting at the price discovery stage. The intuition for this is as follows. First, it is not beneficial to exaggerate costs, as it harms the prospect of progressing to the second stage. Second, the reason why it is not beneficial to understate costs follows the logic of King Solomon's Dilemma, as formalized by Glazer and Ma (1989): if a supplier claims to be more efficient than he really is, he risks to compete in the contest stage under the conditions of his report. The contest stage depends on his report through the endogenous reserve price. This implies that reporting a price below costs risks ending up with negative profits. Notice that truthful reporting at the price discovery stage requires that the organizer can commit to ignore the information revealed during the course of the auction.<sup>24</sup> This is not unrealistic. Lengwiler and Wolfstetter (2006) describe an electronic bid submission system

<sup>22</sup> In Alcalde and Dahm (2016) we analyse a simultaneous-move game of complete information and show that there is a multiplicity of undominated Nash equilibria in each of which the two lowest cost providers are assigned shares. We are grateful to a referee for the observation that the two-stage process of the present section can be understood as selecting one of these equilibria.

<sup>23</sup> We assume that the (initial) exogenous reserve price  $b$  is not very competitive, so that condition (3) holds. This allows us to show how the reserve price can be refined through the procurement procedure, so that procurement becomes more competitive without deterring entry. Subsection 4.4 explains that, since the buyer does not know the costs of providers, adjusting the reserve price based on prior information risks deterring participation. Condition (3) excludes equalities for simplicity. A situation in which  $c_1 = c_2$ , for example, is less interesting for us, as then the trade-off between expenditure minimization and dual sourcing disappears.

<sup>24</sup> A Vickrey auction also has the feature that the final price might be very different from the willingness to pay (or cost in our setting) revealed during the auction. Indeed, Hoffman (2011) describes three

that makes it impossible to replace a bid without leaving a trace. With such a system in place the buyer cannot replace the third lowest report at the price discovery stage with a more competitive price that is high enough not to affect the selection of suppliers for the second stage.<sup>25</sup>

We formalize this two-stage game as follows. We start with some notation. Using Definition 1, for a given vector of prices  $P = (p_1, \dots, p_i, \dots, p_n)$ , define  $A(P) = \{i: p_i < \hat{t}(P)\}$ , and  $B(P) = \{i: p_i = \hat{t}(P)\}$ . Lastly, similar to Glazer and Ma (1989, p. 225) we introduce a parameter  $\epsilon > 0$ . The role of this fixed parameter will become clear shortly, so at this point we only remark that  $\epsilon$  is determined by the buyer as part of the procurement procedure and that for interpretative purposes we think of this number as being very small. More precisely, we assume that such that  $\epsilon < |c_i - c_j|$  for any pair of suppliers  $i \neq j$ .

**Definition 2** The CPA with entry game  $\Gamma^E$  is a two-stage game. The  $n$  suppliers constitute the set of players. At the price discovery stage, each provider reports a price  $p_i \in [0, b]$ . Given the reported prices,  $P = (p_1, \dots, p_i, \dots, p_n)$ , two suppliers advance to the contest stage, according to the rules below. At the contest stage, both providers revise their price selecting

$$r_i \in [0, \hat{t}(P)] . \quad (4)$$

Based on these revised prices procurement shares follow

$$S_i^E(P^E) = \varphi_i^{GCPA}(P^E | \hat{t}(P) + \epsilon) ,$$

where  $P^E$  is the vector in which the entry at position  $h$  is  $r_h$  if  $h$  is one of the suppliers participating in the contest stage, and  $p_h^E = p_h + \epsilon$  otherwise.<sup>26</sup> His profit is thus given by

$$\Pi_i^E(P^E) = S_i^E(P^E) (p_i^E - c_i) , \quad (5)$$

where  $p_i^E$  is the entry at position  $i$  in the vector  $P^E$ .

To conclude the description of  $\Gamma^E$  we explain now how the competitors at the contest stage are selected.

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New Zealand auctions where bidders paid far less than their bid price, including the radio spectrum in 1990. For the latter auction it was revealed that, because of the second-price design, a bidder only paid NZ\$401,000 even though his willingness to pay was NZ\$2,371,000.

<sup>25</sup>Implicitly we also assume that suppliers are prevented from colluding with each other.

<sup>26</sup>Notice that the revised reserve price exceeds the upper bound of the prices at the contest stage by  $\epsilon > 0$ . This is a technical condition needed to apply Theorem 2 in Alcalde and Dahm (2013) that allows us to conclude that the contest stage is dominance solvable. It also guarantees that the shares in the contest stage are strictly positive. Nevertheless, given that  $\epsilon$  is assumed to be very small, for interpretative purposes we will think of the revised reserve price as coinciding with the third most efficient price report of the price discovery stage.

- (a) If  $A(P)$  has two elements, then the competitors at the contest stage are the suppliers in  $A(P)$ .
- (b) If  $A(P)$  is a singleton, then one of the competitors at the contest stage is the supplier in  $A(P)$ . The other provider is selected with equal probability from the suppliers in  $B(P)$ .
- (c) If  $A(P)$  is empty, then the two competitors at the contest stage are selected with equal probability from the suppliers in  $B(P)$ .

In our analysis of  $\Gamma^E$  we focus on Subgame Perfect Nash Equilibria (SPNE henceforth), applying a backward induction argument. Similarly to, for example, Baron and Kalai (1993) or Austen-Smith and Banks (2005, Section 4.1), we apply a notion of weak-stage dominance to the price discovery stage. In order to define such a notion for the first stage of  $\Gamma^E$ , however, it is not necessary to assume that providers anticipate equilibrium play in the second stage. More precisely, the following observations are sufficient. Given a price vector  $P$  from the price discovery stage and conditional on competing at the contest stage, a provider  $i$  anticipates the following about the contest stage, independently with whom he competes and what price his rival sets.

**Observation 1** At the contest stage each provider  $i$  anticipates:

- (a) If  $c_i > \hat{t}(P)$ , then any  $r_i \leq \hat{t}(P)$  implies that the provider receives a strictly positive share at a price strictly below cost, and obtains a strictly negative profit.
- (b) If  $c_i = \hat{t}(P)$ , then any  $r_i < \hat{t}(P)$  again implies losses, and the best the supplier can do is setting  $r_i = \hat{t}(P)$ , which guarantees zero profits.
- (c) If  $c_i < \hat{t}(P)$ , then setting  $r_i = (c_i + \hat{t}(P))/2$  yields a strictly positive share at a price exceeding costs, and thus strictly positive profits (even though this might not be the equilibrium price in the anticipated subgame).

Given that providers anticipate Observation 1, we apply the usual notion of weak dominance to the first stage. That is, for provider  $i$  given, we say that price report  $p_i^*$  weakly dominates  $p_i'$  at the price discovery stage whenever for all price report vectors  $P_{-i} = (p_j)_{j \neq i}$  of his rivals  $\Pi_i(p_i^*, P_{-i}) \geq \Pi_i(p_i', P_{-i})$  holds, with strict inequality for some  $P_{-i}$ . A price report that weakly dominates any other price report is said to be weakly dominant. We say that an equilibrium for  $\Gamma^E$  is a weakly-stage dominant equilibrium, if each provider selects a weakly dominant price report at the price discovery stage. The following theorem describes the unique equilibrium by showing that the truthful price report  $p_i = c_i$  is weakly dominant, which allows to apply the results from Alcalde and Dahm (2013) to the second stage.

**Theorem 1** Assume that providers select weakly dominant price reports at the price discovery stage. Then,  $\Gamma^E$  has a unique SPNE which is fully characterized by (1) suppliers reporting their marginal cost truthfully at the price discovery stage and (2) the contest stage being dominance solvable.

In addition, the buyer's procurement expenditures are lower than  $c_2$  whenever

$$\frac{c_2 - c_1}{c_3 + \epsilon - c_2} > \left( \frac{13}{8} + \frac{5}{8}\sqrt{17} \right) \approx 4.20. \quad (6)$$

The proof of Theorem 1 is relegated to Appendix A.1.

Notice that condition (6) improves upon condition (2), as  $c_3 + \epsilon < b$ . This shows that the existence of a competitive pool of potential suppliers and endogenizing the reserve price through the two-stage procedure  $\Gamma^E$  is beneficial for the buyer, compared to the *Contested Procurement Auction* with only two suppliers.

We conclude this section by pointing out that  $\Gamma^E$  also admits equilibria in which providers use a weakly dominated price report at the price discovery stage. This, however, is not surprising. It is well known that standard Vickrey auctions also admit equilibria in which suppliers employ a weakly dominated strategy.

**Example 1** There are three suppliers with costs  $C = (50, 100, 110)$  and the buyer's budget constraint is  $b = 150$ .

Firstly, consider a standard sealed-bid second-price auction for sole sourcing among the two most efficient suppliers. Notice that the price pair  $(100, 50)$  is a Nash equilibrium in which both providers use a weakly dominated strategy and the most efficient supplier is not selected.

Secondly, consider a sealed-bid Vickrey auction for dual sourcing among all three providers, in which the two winning suppliers provide their shares at the highest price. Suppose winners obtain equal shares. Notice that the price vector  $(100, 110, 100)$  is a Nash equilibrium in which all providers use a weakly dominated strategy and the second most efficient supplier is not selected.

Lastly, consider  $\Gamma^E$ . On one hand, notice that the first stage price report triplet  $(100, x, 100)$  with  $x \in [110, b]$  is part of an equilibrium in which all providers use a weakly dominated price report at the price discovery stage and the second most efficient supplier does not proceed to the contest stage.<sup>27</sup> On the other hand, the focus on weakly

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<sup>27</sup>Notice that the construction of such an equilibrium requires in addition to the use of weakly dominated strategies the following two features that we find unreasonable. First, supplier 3 must 'pre-empt' provider 2 by reporting a price of at most  $c_2$ , as otherwise provider 2 could profitably deviate and lower his report sufficiently to proceed to the contest stage. Second, supplier 2 must make this 'pre-emption' profitable for provider 3 by fixing the reserve price high enough. In other words, supplier 2 can avoid the coordination on such an equilibrium by managing the beliefs of his rivals in such a way that they are certain that he reports a price strictly below  $c_3$ .

undominated price reports yields –as in Vickrey auctions– a unique prediction. The reserve price is revised to  $\hat{t}(P) = 110$ , providers 1 and 2 proceed to the contest stage, and the final provision prices (as  $\epsilon \rightarrow 0$ ) are  $r_1^* = 97.75$  and  $r_2^* = 105.00$ . Procurement shares are 80% for provider 1 and 20% for supplier 2, while expenditures are  $99.23 < c_2$ .

## 4 Extensions

This section considers several extensions of the two-stage process of Section 3. The first three relax the assumption of complete information in different ways. Subsection 4.4 discusses the optimal reserve price and the buyer's objective function.

### 4.1 The two-stage process with private information

In this subsection we introduce private information about costs into the two-stage process of Section 3. We provide conditions on the informational environment under which an equilibrium with truthful reserve price discovery similar to Theorem 1 exists.

We assume that providers have private information about their true costs  $\hat{c}_i$ . Suppose there is a finite number of potential technologies  $T_i$  with  $i = 1, \dots, n$ . The lower the sub-index  $i$ , the more efficient the technology. It is common knowledge that each technology  $T_i$  allows to produce with average costs in some interval  $[\underline{c}_i, \bar{c}_i]$  with  $\bar{c}_i < \underline{c}_{i+1}$ . Note that this hypothesis is similar to that by Landsberger et al. (2001) in that the ranking of costs is common knowledge. To complete the description, let us assume that associated to each technology  $T_i$  there is a continuous probability density function  $f_i: [\underline{c}_i, \bar{c}_i] \rightarrow \mathbb{R}_+$ .

Broadly speaking, the mechanism considered in this subsection is similar to the CPA with entry game in Section 3. Each supplier announces a price at the price discovery stage. These price reports determine the revised reserve price and the identity of the providers competing at the contest stage. The price reports itself, however, are not made public. The suppliers participating at the contest stage share the supply contract based on their revised prices. Since suppliers do not have perfect information about all relevant parameters, the game considered in this subsection differs formally from that introduced in Definition 2. We refer to this game as the *modified CPA with entry game* and denote it by  $\Gamma^{\text{mE}}$ . Our equilibrium concept is Perfect Bayesian Equilibrium (PBE henceforth).

We now introduce a condition under which the analysis of the contest stage becomes tractable. It establishes that

$$\frac{\hat{c}_3 + \epsilon - \underline{c}_2}{\hat{c}_3 + \epsilon - E(c_2)} \leq \frac{\hat{c}_3 + \epsilon - \bar{c}_1}{\hat{c}_3 + \epsilon - \underline{c}_2}, \quad (\text{C1})$$

where

$$E(c_2) = \int_{\underline{c}_2}^{\bar{c}_2} c_2 f_2(c_2) dc_2$$

is the expected value of  $c_2$ . Appendix A.2 offers a discussion of the difficulties that arise in settings in which Condition (C1) does not hold. Notice, however, that Condition (C1) is more likely to hold, the more precise the information about the cost of provider 2 is. In particular, when  $|\bar{c}_2 - \underline{c}_2| \rightarrow 0$ , Condition (C1) always holds.

With these considerations in mind we can establish the following result.<sup>28</sup>

**Theorem 2** Assume that providers select weakly dominant price reports at the price discovery stage. Then, when Condition (C1) holds, the modified CPA with entry game  $\Gamma^{\text{mE}}$  has a PBE, which is described as follows. First, at the price discovery stage each provider reports  $p_i = \hat{c}_i$ ; and second, the Bayesian equilibrium for the contest stage is described by

$$\begin{aligned} r_2(c_2) &= \frac{\hat{c}_3 + \epsilon + c_2}{2} \\ r_1(c_1) &= \hat{c}_3 + \epsilon - \sqrt{\frac{(\hat{c}_3 + \epsilon - c_1)(\hat{c}_3 + \epsilon - E(c_2))}{4}} \end{aligned}$$

Appendix A.2 proposes a formal proof for Theorem 2.

The intuition for Theorem 2 is as follows. As a result of the price discovery stage only the revised reserve price and the identity of the suppliers participating at the contest stage are revealed. In particular, the prices selected are revealed to the buyer but not to the rivals. As a consequence, the arguments provided in Observation 1 are still valid for the game  $\Gamma^{\text{mE}}$  and each provider selects  $p_i = \hat{c}_i$  at the price discovery stage, revealing his true marginal cost to the buyer (but not to the other providers). Thus, providers 1 and 2 advance to the contest stage and the revised budget constraint becomes  $\hat{b} = \hat{t}(P) + \epsilon = \hat{c}_3 + \epsilon$ . The role of Condition (C1) is to make sure that behaviour at the contest stage is consistent. Since suppliers know that  $\hat{c}_1 < \hat{c}_2$ , in the equilibrium constructed, provider 2 behaves as if his revised price is higher than the one selected by provider 1. Consequently, his revised price is bounded by

$$\frac{\hat{c}_3 + \epsilon + \underline{c}_2}{2} \leq r_2 = \frac{\hat{c}_3 + \epsilon + \hat{c}_2}{2} \leq \frac{\hat{c}_3 + \epsilon + \bar{c}_2}{2}.$$

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<sup>28</sup>We conjecture that the Bayesian equilibrium for the contest stage, described in Theorem 2 below, is unique. Uniqueness would imply that the existence of a small amount of uncertainty about the costs of rivals prevents providers to select weakly dominated price reports at the price discovery stage in the context of Example 1. A similar result is true in the Vickrey auction with binding reserve price, see Blume and Heidhues (2004).

When Condition (C1) holds, provider 1's best reply to supplier 2's optimal revised price is indeed lower than  $r_2(c_2)$  for each  $c_2 \in [c_2, \bar{c}_2]$ . Example 2 in Appendix A.2 illustrates the consequences and computational difficulties that arise when Condition (C1) does not hold.

## 4.2 The CPA with entry, incomplete information, and loser regret

In the modified CPA with entry game of the previous subsection, the informational environment is restricted and the price reports at the price discovery stage are not made public. In this subsection we consider a more general informational setting and allow for all information to be revealed. We explain that when loser regret is an important enough consideration at the discovery stage, an equilibrium with truthful reserve price discovery similar to Theorem 1 exists.

Consider that each provider's cost is private information, that the price reports at the price discovery stage are made public, and that similar to Engelbrecht-Wiggans (1989), Engelbrecht-Wiggans and Katok (2007, 2008) or Filiz-Ozbay and Ozbay (2007) suppliers are loss (or regret) averse. In the two-stage process of Section 3 two types of regret can appear.<sup>29</sup> On one hand, supplier  $i$  might have loser regret, because a pair of suppliers  $j$  and  $k$  report prices, say  $p_j < p_k$ , at the discovery stage and advance to the contest stage, even though  $c_i < p_k$  holds. Hence, provider  $i$  missed out on a positive profit that he could have earned by reporting a lower price and advancing to the contest stage. On the other hand, supplier  $i$  might have winner regret, because he advances to the contest stage but all permissible prices are lower than his cost. In such a case provider  $i$  is certain to make losses that could have been avoided by reporting a higher price at the discovery stage. In Appendix A.3 we formalize these considerations. We show that when regret is an important enough consideration at the discovery stage, then there is a symmetric equilibrium in which suppliers report their marginal costs truthfully at the first stage and the contest stage is played under complete information.

## 4.3 A reverse English auction of incomplete information

The previous two subsections either restrict the informational environment or assume loser regret in order to introduce private information in the two-stage process of Section 3. In this subsection we dispense with both assumptions but vary the auction procedure employed. Again, we derive an equilibrium with truthful reserve price discovery similar to Theorem 1.

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<sup>29</sup>In the aforementioned (forward) first-price sealed-bid auctions two related types of regret appear. Winner regret refers to a winner paying more than the second highest bid, while loser regret appears when a loser has a higher value than what the winner paid for the object.



More precisely, we follow Edelman et al. (2007) and Alcalde and Dahm (2013) and consider a variant of a reverse English auction with rules that resemble the two-stage process of Section 3. We assume that each supplier only has (private) information about his own costs, but does not know the costs of his rivals. We analyse a simple continuous-time version of our procedure that is easily implementable through the increased ability to communicate in real time via the Internet. During the course of the auction, information about rivals is not needed, because all the relevant information is revealed. In particular, discarded suppliers reveal their costs truthfully. As a result, at the unique equilibrium the procurement shares and profits of providers coincide with those in the complete information environment of Theorem 1.

Since the procedure is based on an *electronic reverse auction*, we refer to it as ‘Electronic Procurement’ mechanism. We describe the corresponding game  $\Gamma^{\text{EP}}$  as follows.

**Definition 3** The ‘Electronic Procurement’ game  $\Gamma^{\text{EP}}$  is a continuous-time mechanism. The  $n$  suppliers constitute the set of players. At each moment  $m \in [0, 1]$  all providers simultaneously select their messages (or actions)  $a_{im} \in \{0, 1\}$ , where 0 is interpreted as ‘continuing’ and 1 as ‘stopping’. We denote by  $m_i$  the first moment at which  $i$  selects  $a_{im_i} = 1$ , that is,  $a_{im_i} = 1$  and  $a_{im} = 0$  for each  $m < m_i$ .

Given the sequence of actions of providers, supplier  $i$ ’s price is given by

$$p_i = (1 - m_i) b, \quad (7)$$

where  $b$  is the initial (exogenous) reserve price. At  $m = 1$  the auction closes and we interpret this as all suppliers choosing at this moment message 1.<sup>30</sup> At the conclusion of the auction all prices are hence determined in an indirect way through equation (7) and each supplier’s procurement share follows

$$S_i^{\text{EP}}(P) = \varphi_i^{\text{GCPA}}(P | \hat{t}(P)), \quad (8)$$

where  $\hat{t}(P)$  was introduced in Definition 1. His profit is thus given by

$$\Pi_i^{\text{EP}}(P) = S_i^{\text{EP}}(P) (p_i - c_i). \quad (9)$$

Given the definition of the game  $\Gamma^{\text{EP}}$ , we formalize now the assumptions on observability and the informational structure, which are common knowledge. Following Edelman et al. (2007) we assume that average costs  $c_i$  are independently drawn from a continuous distribution  $F(\cdot)$  on  $[0, b]$  with a continuous density function  $f(\cdot)$  that is positive everywhere on  $(0, b)$ . Initially, at moment  $m = 0$ , each provider  $i$  knows only his

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<sup>30</sup>That is, we use the convention that  $p_i = 0$  when  $a_{im} = 0$  for each  $m \in [0, 1]$ . Notice also that once supplier  $i$  selects  $a_{im} = 1$  his price is determined. Therefore, his actions at any  $m > m_i$  are inconsequential.

own costs  $c_i$ , the distribution of other firms' costs, and the initial (exogenous) reserve price  $b$ . In particular, we suppose that suppliers do not know their rivals' costs. During the course of the auction, however, additional information about providers is revealed, because the message of each supplier at each moment in time is observable by all other providers. Hence, at each moment  $m \in [0, 1]$  each provider  $i$  knows the messages selected by all suppliers at any  $m' < m$ . We represent supplier  $i$ 's relevant information at any  $m \in [0, 1]$  by

$$I_i(m) = \left( c_i, L(m), (m_j)_{j \notin L(m)} \right), \quad (10)$$

where  $L(m)$  denotes the set of providers whose prices are still undetermined; i.e.  $j \in L(m)$  if for each  $m' < m$ ,  $a_{jm'} = 0$ . The last entry in (10), the collection of  $m_j$  for  $j \notin L(m)$ , are the drop out decisions of the remaining providers, which are observable and determine the prices of these suppliers.

A strategy for provider  $i$  prescribes for each  $\tilde{m} \in [0, 1)$  an action  $a_{i\tilde{m}}$  that depends both on  $\tilde{m}$ , which determines the current price, and on the provider's current information, which is collected in  $I_i(\tilde{m})$ . Consider the strategy described by

$$a^* = \begin{cases} 0 & \text{if } \alpha(I_i(\tilde{m})) < (1 - \tilde{m})b \\ 1 & \text{otherwise} \end{cases}, \quad (11)$$

where

$$\alpha(I_i(\tilde{m})) = \begin{cases} c_i & \text{if } |L(\tilde{m})| \geq 3 \\ \frac{\hat{t}(P) + c_i}{2} & \text{if } |L(\tilde{m})| = 2 \\ \hat{t}(P) - \sqrt{\frac{(\hat{t}(P) - c_i)(\hat{t}(P) - \delta)}{2}} & \text{otherwise} \end{cases}, \quad (12)$$

and  $\delta$  is the second-lowest price; i.e.  $\delta = (1 - \bar{m})b$ , where  $\bar{m}$  is such that  $L(\bar{m})$  has two providers and, for any  $m$  with  $\bar{m} < m \leq \tilde{m}$ ,  $L(m)$  is a singleton. The strategy  $a^*$  requires the provider to remain active until the price drops below a certain threshold. This threshold depends on the number of suppliers remaining in the auction. If more than two suppliers remain active, then the provider waits until his marginal cost is reached. If all but one other supplier have dropped out, then the provider shades his bid by dropping out earlier. These drop out decisions mimic the revised provision prices in the contest stage of Theorem 1. We have the following result.

**Proposition 1** In game  $\Gamma^{\text{EP}}$ , it is a weakly dominant strategy to follow  $a^*$ .

The proof of Proposition 1 is relegated to Appendix A.4.

The intuition why the strategy  $a^*$  is weakly dominant is simple. First, as in a standard English auction, it cannot be optimal to continue when the price falls below marginal costs, as this can only result in a loss. Also, dropping out before the price reaches marginal costs risks forgoing potential gains and cannot be optimal either. Second, once only two providers are left, the key observation is the following. The supplier dropping out first is certain to submit the second lowest price, and the optimal second lowest price does not depend on the lowest price. Third, once only one supplier is left, he observes the drop out decisions and thus prices of all other providers. He shades his bid compared to what would be his optimal second lowest price, resolving optimally the trade-off between procurement share and mark up.

When all providers follow the weakly dominant strategy  $a^*$  each supplier  $i > 2$  reveals his costs truthfully by choosing  $p_i = c_i$ . This implies that the revised reserve price coincides with  $c_3$ . The last two suppliers shade their bid and drop out earlier than  $c_i$ . Since these drop out decisions mimic the revised provision prices in the contest stage of Theorem 1, in the unique equilibrium the procurement shares and profits of providers coincide with those in the complete information environment of Theorem 1. We record this with the following result; the proof is immediate taking into account Proposition 1 and Theorem 1.

**Theorem 3** The weakly dominant strategies of providers in  $\Gamma^{\text{EP}}$  induce the following prices

$$p_i = \begin{cases} c_i & \text{if } i \geq 3 \\ \frac{c_3 + c_2}{2} & \text{if } i = 2 \\ c_3 - \sqrt{\frac{(c_3 - c_2)(c_3 - c_1)}{4}} & \text{if } i = 1 \end{cases} \quad (13)$$

Moreover, if

$$\frac{c_2 - c_1}{c_3 - c_2} > \left( \frac{13}{8} + \frac{5}{8}\sqrt{17} \right) \approx 4.20,$$

then the buyer's procurement expenditures are lower than  $c_2$ .

Notice that the payoff equivalence between the games  $\Gamma^{\text{EP}}$  and  $\Gamma^{\text{E}}$  is very similar to the one between the simultaneous-move game of complete and the extensive-form game of incomplete information in Edelman et al. (2007). One difference, however, is that their equilibrium of the extensive-form game is not in weakly dominant strategies.

The fact that providers have a weakly dominant strategy implies that the procurement procedure can be used under different assumptions of what providers know about

each others' marginal costs, provided the assumption of private values holds. If, however, there is interdependence of providers' costs, the drop out decisions of rivals might reveal information about a supplier's cost and it is not weakly dominant to follow the strategy  $a^*$ .

## 4.4 The Buyer's Objectives and the Optimal Reserve Price

The analysis so far has made the benchmark assumption that the buyer's only objective is to minimize procurement costs. This simplification has two advantages. First, it makes our result, that the buyer might prefer to use our procedure rather than a standard Vickrey auction for sole sourcing, more surprising than the alternative assumption that dual sourcing has value by itself. We are able to follow this approach, because under condition (6) in Theorem 1, our procedure induces very competitive procurement, so that the trade-off between expenditure minimization and dual sourcing disappears. Second, it offers a clear benchmark of comparison. Alternatively, we could postulate an objective function for the buyer that specifies how she trades-off expenditure minimization and the benefits of dual sourcing. But doing so would yield results that depend on the specific formalization of the buyer's objectives. Since valuing dual sourcing would provide additional incentives for dual sourcing, condition (6) in Theorem 1 would be relaxed and our procedure would become more attractive.

One important reason for why a buyer might value dual sourcing by itself is that it helps to manage the risk that a supplier cannot fulfil the contractual obligations. This causes delays and in the extreme the buyer might be unable to obtain provision.<sup>31</sup> Consider a buyer who values dual sourcing by itself, because it reduces the risk of having no provision. In such a situation our procedure with endogenous reserve price through price discovery is very attractive, because the buyer 'can play it safe', set a high initial reserve price, and refine it through the procurement procedure in such a way that entry is not deterred and provision is guaranteed. Alternatively, the buyer could use our procedure but set the reserve price optimally, based on prior information. Besides having the practical disadvantages described in the Introduction, doing so risks setting the reserve price too low and to deter participation, resulting in high procurement cost

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<sup>31</sup>Engel et al. (2006) discuss that bankruptcy costs of providers cause important costs for the buyer and how sourcing to more than one contractor can manage this risk. Another reason for why a supplier might not be able to fulfil the contractual obligations is that the regulator might suspend the license. In the autumn of 2004, the U.S. experienced a severe influenza vaccine shortage, because one of two suppliers (Chiron) failed to produce the expected half of the necessary vaccines. For an account and a discussion that such a risk can be mitigated by increasing the number of contractors, see the editorial "An Influenza Vaccine Debacle" in the New York Times, October 20, 2004, <http://www.nytimes.com/2004/10/20/opinion/an-influenza-vaccine-debacle.html?mcubz=1>, accessed on 05/08/2018.

and sole sourcing or no provision at all. Consequently, when it is sufficiently important to avoid failure of provision, the buyer will prefer to refine the reserve price through price discovery.

## 5 Concluding Remarks

This paper has proposed a new procurement procedure for dual sourcing. Commonly employed dual sourcing strategies fix procurement shares and the reserve price exogenously. In contrast, our procedure uses the bids of suppliers in order to endogenize both the allocation of shares and the reserve price. We have formalised this idea in different ways and shown that in equilibrium providers reveal their costs truthfully, so that the two most competitive suppliers are awarded contracts. When discarded suppliers are competitive enough, the procedure not only allows taking advantage of dual sourcing but also generates lower procurement expenditures than a standard auction for sole (or dual) sourcing. Moreover, the procedure can be used under different assumptions of what providers know about each others' costs, provided the assumption of private values holds.

There are several interesting ways in which our analysis might be extended. In particular, our assignment rule for shares assumes the elasticity of a supplier's procurement share with respect to his price to be one. Following Alcalde and Dahm (2013) this could be generalized to other values of the elasticity. The results in our earlier paper suggest that under such a generalization truthful revelation of marginal costs of discarded bids still occurs in equilibrium. Theorem 3 in Alcalde and Dahm (2013) implies then that the buyer can choose the elasticity in such a way that procurement expenditures are lower than in a Vickrey auction for sole sourcing, even when in the setting of the present paper (with unit elasticity) this is not possible. This shows that different assignment rules for shares might yield further interesting results.

We also assumed that the providers' marginal costs are constant. Introducing economies of scale poses a challenge, as it should make it more difficult to reconcile the aims of expenditure minimization and having more than one provider. But the latter can still be desirable. Scherer (2007), for instance, analyses for influenza vaccines the trade-off between economies of scale and protection against stochastic shortage risk through having more than one provider. He concludes that for plausible scenarios sole sourcing is not optimal. We leave these extensions for future work.

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## A Appendix

### A.1 Proof of Theorem 1

This Appendix provides a proof of Theorem 1. We first show that at the price discovery stage for each supplier  $i$  the truthful price report  $p_i = c_i$  is weakly dominant. To do so, we proceed in two steps (Lemma 1 and Lemma 2). We turn then to the derivation of the threshold in condition (6).

**Lemma 1** For each supplier  $i$  any price report  $p_i < c_i$  is weakly dominated by  $p'_i = c_i$ .

**Proof.** Consider a given supplier, say  $i$ , a price report vector  $P_{-i} = (p_j)_{j \neq i}$  of his rivals, and let  $p_{(1)}$  and  $p_{(2)}$  be the lowest competing prices. More precisely, let  $p_{(1)} \leq p_{(2)} \leq p_j$  for each  $j \notin \{(1), (2), i\}$ . We assume  $0 \leq p_i < p'_i = c_i$  and consider the following cases, which exhaust all the possibilities.

- (a)  $p'_i < p_{(2)}$ . Then  $i \in A(P)$  and  $i \in A(P')$ , so that with both reports  $i$  proceeds to the second stage. Moreover,  $\hat{t}(P) = \hat{t}(P')$ , and hence  $\Pi_i(p_i, P_{-i}) = \Pi_i(p'_i, P_{-i})$ .
- (b)  $p_{(2)} < p_i$ . Then with both reports provider  $i$  does not proceed to the second stage and obtains zero profits.
- (c)  $p_i \leq p_{(2)} < p'_i$ . In this case provider  $i$  proceeds with some positive probability to the second stage when reporting  $p_i$ , but does not proceed when reporting  $p'_i$ . In the latter case his profits are zero. In the former case  $i$ 's profit is strictly negative, as  $c_i > \hat{t}(P)$ .
- (d)  $p_{(2)} = p'_i$ . Then  $i \in A(P)$  and  $i \in B(P')$ , so that reporting  $p_i$  supplier  $i$  is certain to proceed to the second stage, while reporting  $p'_i$  supplier  $i$  only proceeds with a positive probability to the second stage. Since however  $\hat{t}(P') = \hat{t}(P) = c_i$ , profits are zero when proceeding to the second stage and hence  $\Pi_i(p_i, P_{-i}) = \Pi_i(p'_i, P_{-i})$ .

Note that in all cases it is weakly better to report  $p'_i$  rather than  $p_i$ , with a strict preference in case (c). Hence, any price report  $p_i < c_i$  is weakly dominated by  $p'_i = c_i$ .

■

**Lemma 2** For each supplier  $i$  any price report  $p_i > c_i$  is weakly dominated by  $p'_i = c_i$ .

**Proof.** Consider a given supplier, say  $i$ , and a price report vector  $P_{-i} = (p_j)_{j \neq i}$  of his rivals, and let  $p_{(1)}$  and  $p_{(2)}$  be the lowest competing prices. More precisely, let  $p_{(1)} \leq p_{(2)} \leq p_j$  for each  $j \notin \{(1), (2), i\}$ . We assume  $c_i = p'_i < p_i$  and consider the following cases, which exhaust all the possibilities.

- (a)  $p'_i < p_i < p_{(2)}$ . Then  $i \in A(P)$  and  $i \in A(P')$ , so that with both reports  $i$  proceeds to the second stage. Moreover,  $\hat{t}(P) = \hat{t}(P')$ , and hence  $\Pi_i(p_i, P_{-i}) = \Pi_i(p'_i, P_{-i})$ .
- (b)  $p'_i < p_{(2)} < p_i$ . In this case reporting  $p'_i$  supplier  $i$  proceeds to the second stage and obtains a strictly positive profit, as  $\hat{t}(P') > c_i$ . On the other hand, reporting  $p_i$  his profits are zero, because he does not proceed to the second stage.
- (c)  $p'_i < p_{(2)} = p_i$ . Then  $c_i < \hat{t}(P') = \hat{t}(P)$ , and provider  $i$  strictly prefers to report  $p'_i$ , as reporting  $p_i$  he does not always proceed to the second stage.
- (d)  $p_{(2)} < p'_i < p_i$ . Then with both reports provider  $i$  does not proceed to the second stage and obtains zero profits.
- (e)  $p_{(2)} = p'_i < p_i$ . Then, when reporting  $p'_i$  supplier  $i$  has a positive probability of proceeding to the second stage in which case his profit is zero, as  $\hat{t}(P') = c_i$ . Reporting  $p_i$  his profits are also zero, because he does not proceed to the second stage.

Note that in all cases it is weakly better to report  $p'_i$  rather than  $p_i$ , with a strict preference in cases (b) and (c). Hence, any price report  $p_i > c_i$  is weakly dominated by  $p'_i = c_i$ .

■

To find the threshold described in equation (6), under which the buyer's procurement expenditures do not exceed  $c_2$ , we proceed as follows. Lemmata 1 and 2 above imply that for each supplier  $i$  report  $p_i = c_i$  weakly dominates every other report at the price discovery stage. Thus,  $\hat{t}(P) = c_3$ , and providers 1 and 2 compete at the contest stage. Each of them has to select a revised price  $r_i \in [0, c_3]$ , while procurement shares are assigned based on the slightly higher reserve price  $\hat{b} = c_3 + \epsilon$ , as explained in footnote 26. Theorems 1 and 2 in Alcalde and Dahm (2013) imply that this subgame is dominance solvable and has a unique equilibrium.

This unique equilibrium (for the contest stage), say  $r^* = (r_1^*, r_2^*)$ , is described by

$$\begin{aligned}
 r_1^* &= \hat{b} - \sqrt{\frac{(\hat{b} - c_1)(\hat{b} - c_2)}{4}} \\
 r_2^* &= \frac{\hat{b} + c_2}{2}
 \end{aligned}$$

Therefore, the provision share assigned to provider 2 is

$$S_2^E = \frac{\hat{b} - r_2^*}{2(\hat{b} - r_1^*)} = \sqrt{\frac{\hat{b} - c_2}{4(\hat{b} - c_1)}} \quad (14)$$

and supplier 1 obtains the share  $S_1^E = 1 - S_2^E$ .

Hence, the buyer's equilibrium expenditure is

$$\begin{aligned}
C(\hat{b}|r^*) &= S_1^E r_1^* + S_2^E r_2^* &= \\
&= r_1^* - (r_1^* - r_2^*) S_2^E &= \\
&= \hat{b} - \sqrt{\frac{(\hat{b} - c_1)(\hat{b} - c_2)}{4}} - \frac{\hat{b} - c_2}{4} \sqrt{\frac{\hat{b} - c_2}{\hat{b} - c_1}} + \frac{\hat{b} - c_2}{4} &= \\
&= \hat{b} - \sqrt{\frac{(\hat{b} - c_1)(\hat{b} - c_2)}{4}} + \frac{\hat{b} - c_2}{4} \left(1 - \sqrt{\frac{\hat{b} - c_2}{\hat{b} - c_1}}\right).
\end{aligned}$$

Recall that our aim is to find a condition under which  $C(\hat{b}|r^*) \leq c_2$ . Taking into account the general expression for  $C(\hat{b}|r^*)$  above, we have that

$$\begin{aligned}
C(\hat{b}|r^*) &\leq c_2 &\Leftrightarrow \\
\hat{b} - c_2 &\leq \sqrt{\frac{(\hat{b} - c_1)(\hat{b} - c_2)}{4}} - \frac{\hat{b} - c_2}{4} \left(1 - \sqrt{\frac{\hat{b} - c_2}{\hat{b} - c_1}}\right) &\Leftrightarrow \\
\frac{5}{4}(\hat{b} - c_2) &\leq \sqrt{\frac{(\hat{b} - c_1)(\hat{b} - c_2)}{4}} + \frac{\hat{b} - c_2}{4} \sqrt{\frac{\hat{b} - c_2}{\hat{b} - c_1}} &\Leftrightarrow \\
5 - \sqrt{\frac{\hat{b} - c_2}{\hat{b} - c_1}} &\leq \sqrt{\frac{4(\hat{b} - c_1)}{\hat{b} - c_2}} &\Leftrightarrow \\
0 &\leq 2 \left(\frac{\hat{b} - c_1}{\hat{b} - c_2}\right) - 5 \sqrt{\frac{\hat{b} - c_1}{\hat{b} - c_2}} + 1 &\Leftrightarrow \\
\frac{\hat{b} - c_1}{\hat{b} - c_2} &\geq \left(\frac{5}{4} + \frac{1}{4}\sqrt{17}\right)^2 = \frac{21}{8} + \frac{5}{8}\sqrt{17}.
\end{aligned}$$

Since

$$\frac{\hat{b} - c_1}{\hat{b} - c_2} = \frac{c_2 - c_1}{\hat{b} - c_2} + 1,$$

it follows that

$$C(\hat{b}|r^*) \leq c_2 \Leftrightarrow \frac{c_2 - c_1}{\hat{b} - c_2} \geq \frac{13}{8} + \frac{5}{8}\sqrt{17}.$$

This completes the proof of Theorem 1.

## A.2 Proof of Theorem 2

This Appendix deals with the analysis of the modified CPA with entry game from Subsection 4.1. We first discuss the difficulties of computing an equilibrium, if it exists, when Condition (C1) does not hold. Then we provide a formal proof for Theorem 2.

As explained in the main text, the arguments in Appendix A.1 can be adapted to show that each provider has a weakly dominant price report at the price discovery stage, namely to truthfully reveal his cost  $p_i(c_i) = c_i$ . This allows to consider the contest stage subgame as a standard private values setting, in which the players are suppliers 1 and 2. The cost of each supplier is drawn from the cumulative distribution  $F_i$  with support  $[\underline{c}_i, \bar{c}_i]$ . Associated to each  $F_i$  there is a positive continuous density  $f_i: [\underline{c}_i, \bar{c}_i] \rightarrow \mathbb{R}_+$ .

A strategy for provider  $i = 1, 2$  at the contest stage subgame, is a function  $r_i: [\underline{c}_i, \bar{c}_i] \rightarrow [0, \hat{b}]$  associating a revised price to each value of his cost.

At the contest stage, given the revised price  $r_i \in [0, \hat{t}(P)]$  submitted by each provider, provider  $i$ 's payoffs follow the expression

$$\Pi_i^{mE} = \begin{cases} \frac{\hat{b} - r_i}{2(\hat{b} - r_j)} [r_i - c_i] & \text{if } r_i \geq r_j \\ \left[1 - \frac{\hat{b} - r_j}{2(\hat{b} - r_i)}\right] [r_i - c_i] & \text{otherwise} \end{cases}. \quad (15)$$

Assume that provider  $j$  follows the increasing and differentiable equilibrium strategy  $r_j(c_j)$  and also that provider  $i$ , given the realization of his costs  $c_i$ , submits  $r_i$ . We wish to determine the optimal  $r_i$ .

Supplier  $i$  is the high-cost provider (and thus the first row in equation (15) is relevant) whenever  $r_i \geq r_j(c_j)$ , and is the low-cost provider otherwise. His expected payoff from submitting  $r_i$  is hence

$$\begin{aligned} EU_i(r_i | c_i) &= \int_{\underline{c}_j}^{c_j^*} \frac{\hat{b} - r_i}{2(\hat{b} - r_j(c_j))} [r_i - c_i] f_j(c_j) dc_j + \\ &+ \int_{c_j^*}^{\bar{c}_j} \left[1 - \frac{\hat{b} - r_j(c_j)}{2(\hat{b} - r_i)}\right] [r_i - c_i] f_j(c_j) dc_j \end{aligned}$$

where the threshold  $c_j^* \in [\underline{c}_j, \bar{c}_j]$  satisfies that  $r_i = r_j(c_j^*)$ . If such a value does not exist, we set  $c_j^* = \bar{c}_j$  if  $r_i > r_j(\bar{c}_j)$ , and  $c_j^* = \underline{c}_j$  if  $r_i < r_j(\underline{c}_j)$ .

By maximizing the latter expression we get the first order condition

$$\int_{\underline{c}_j}^{c_j^*} \frac{c_i + \hat{b} - 2r_i}{2(\hat{b} - r_j(c_j))} f_j(c_j) dc_j + \int_{c_j^*}^{\bar{c}_j} \left[ 1 - \frac{(\hat{b} - r_j(c_j))(\hat{b} - c_i)}{2(\hat{b} - r_i)^2} \right] f_j(c_j) dc_j = 0. \quad (16)$$

Notice that for  $r_i = \hat{b}$  both terms on the left hand side of equation (16) are negative, while for  $r_i = c_i$  both terms are positive. Since both terms are continuous functions of  $r_i$ , we can apply Bolzano's Theorem and conclude that a solution to (16) exists. Note also, however, that it is not possible to find this solution analytically. Even assuming symmetry, that is,  $[\underline{c}_1, \bar{c}_1] = [\underline{c}_2, \bar{c}_2]$  and  $F_i = F$  for  $i = 1, 2$ , so that it is natural to focus on a symmetric equilibrium does not simplify the problem sufficiently.

Let us go further and assume that, since  $\bar{c}_1 < \underline{c}_2$ , providers 1 and 2 believe that the realized revised prices should satisfy  $r_1 < r_2$ , which is very intuitive. Supplier 2 is the high-cost provider and expects the first row in (15) to be relevant, while provider 1 is the low-cost provider and expects the second row to be relevant. Supplier 2's first order condition is then given by (16) with  $i = 2, j = 1$  and  $c_1^* = \bar{c}_1$ . It is straightforward to see that

$$r_2(c_2) = \frac{\hat{b} + c_2}{2} \quad (17)$$

solves this equation. On the other hand, provider 1's first order condition is given by (16) with  $i = 1, j = 2$  and  $c_2^* = \underline{c}_2$ . Using (17) in this expression, it can be shown that the first order condition holds for

$$r_1(c_1) = \hat{b} - \sqrt{\frac{(\hat{b} - c_1)(\hat{b} - E(c_2))}{4}}. \quad (18)$$

Notice that provider 1's strategy in (18) is increasing both in  $c_1$  and  $E(c_2)$ . Moreover, replacing  $E(c_2)$  by  $c_1$  we obtain that the revised price is the same function of cost as in (17). This shows that provider 1 optimally shades the price to trade-off the share of provision with the mark-up. In addition, supplier 1's information is imprecise and it might turn out that, for some realizations of random variable  $c_2$ , say  $\hat{c}_2$ ,  $E(c_2) > \hat{c}_2$  holds. In such a situation the revised price is even less competitive than it would be under complete information. As a result of both effects suppliers might revise their prices expecting that the revised prices satisfy  $r_1 < r_2$  but the actual choices violate this condition, because provider 1's price is not competitive enough. We illustrate this possibility with the following example.

**Example 2** Let each  $c_i$  be uniformly distributed on  $[\underline{c}_i, \bar{c}_i]$ , and thus  $f_i(c_i) = (\bar{c}_i - c_i)^{-1}$ . Let  $[\underline{c}_1, \bar{c}_1] = [37, 42]$  and  $[\underline{c}_2, \bar{c}_2] = [43, 49]$ . The revised reserve price is  $\hat{b} = 50$ .<sup>32</sup> The providers' true costs are  $\hat{c}_1 = 41$ ,  $\hat{c}_2 = 43.5$ . Evaluating the prices in (17) and (18) at these costs we see that supplier 1's price is not competitive enough, because  $r_1 = 47$  and  $r_2 = 46.75$ .

This example points out that it is not enough to require that  $\bar{c}_1 < \underline{c}_2$ , so that the agents' conjectures, about how the revised prices are ranked, is fulfilled. In this example  $r_1(c_1) < r_2(c_2) = 46.5$  does not hold and thus supplier 1's first order condition does not simplify as assumed in the above derivation of (18). To make sure that this derivation is valid for all values of costs we need to assume that  $r_1(\bar{c}_1) \leq r_2(\underline{c}_2)$ , which using (17) and (18) can be shown to be equivalent to Condition (C1).

## Proof of Theorem 2

First note that, as previously observed, at the price discovery stage, each provider truthfully reveals (the realization of) his costs, namely  $p_i(c_i) = c_i$ . Assume that the cost for provider 3 is  $\hat{c}_3$ , so that suppliers 1 and 2 compete at the contest stage with the revised reserve price  $\hat{b} = \hat{c}_3 + \epsilon$ . Since we focus on PBE in which all the suppliers select weakly dominant price reports, there is no loss of generality in restricting our analysis to Bayesian equilibria of the contest stage subgame. For each of these providers, let  $r_i: [\underline{c}_i, \bar{c}_i] \rightarrow [0, \hat{b}]$  be his strategy, which associates a revised price to each value of his cost.

Assume that provider 2 with cost  $c_2$  believes that 1's strategy  $r_1(\cdot)$  involves the selection of the lower revised price. Then, selecting the revised price  $x$ , supplier 2's expected utility is

$$EU_2(x | r_1(c_1) \leq x) = \int_{\underline{c}_1}^{\bar{c}_1} \frac{\hat{b} - x}{2(\hat{b} - r(c_1))} [x - c_2] f_1(c_1) dc_1.$$

Maximizing this expression with respect to  $x$  we can derive the following strategy for provider 2

$$r_2(c_2) = \frac{\hat{b} + c_2}{2}.$$

Analogously, assume that provider 1 with cost  $c_1$  believes that supplier 2 proposes the higher revised price. Then, selecting the revised price  $y$ , his expected utility follows

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<sup>32</sup> This revised reserve price arises for example from the price discovery stage if there is a third provider with  $[\underline{c}_3, \bar{c}_3] = [49.5, 51]$ , supplier 3's true cost is  $\hat{c}_3 = 49.9$ , the initial exogenous budget constraint is  $b = 150$ , and the parameter  $\epsilon$  is 0.1. The revised reserve price is hence  $\hat{b} = \hat{t}(P) + \epsilon = \hat{c}_3 + \epsilon = 50$ .

the expression

$$EU_1(y | r_2(c_2) \geq y) = \int_{\underline{c}_2}^{\bar{c}_2} \left[ 1 - \frac{\hat{b} - r_2(c_2)}{2(\hat{b} - y)} \right] [y - c_1] f_2(c_2) dc_2.$$

Maximizing this expression with respect to  $y$ , taking into account that 2 chooses strategy  $r_2(\cdot)$  above, we derive the following strategy for supplier 1

$$r_1(c_1) = \hat{b} - \sqrt{\frac{(\hat{b} - c_1)(\hat{b} - E(c_2))}{4}}.$$

Note that, since  $r_2$  is increasing in  $c_2$ , we have that  $r_2(c_2) \geq r_2(\underline{c}_2)$  for all  $c_2 \in [\underline{c}_2, \bar{c}_2]$ . Similarly, as  $r_1$  is increasing in  $c_1$ , for each  $c_1 \in [\underline{c}_1, \bar{c}_1]$ ,  $r_1(c_1) \leq r_1(\bar{c}_1)$ . A necessary condition for the strategies  $r_i(c_i)$  above to constitute a Bayesian equilibrium is that for each  $c_1$  and any  $c_2$ ,  $r_1(c_1) \leq r_2(c_2)$ . By Condition (C1) we have that

$$\frac{\hat{b} - \underline{c}_2}{\hat{b} - E(c_2)} \leq \frac{\hat{b} - \bar{c}_1}{\hat{b} - \underline{c}_2},$$

which is equivalent to  $r_1(\bar{c}_1) \leq r_2(\underline{c}_2)$ . Therefore, for each  $c_1$  and any  $c_2$ ,  $r_1(c_1) \leq r_1(\bar{c}_1) \leq r_2(\underline{c}_2) \leq r_2(c_2)$ .

Now, assume that the strategies above do not constitute an equilibrium. Therefore, one of the suppliers is not reacting optimally to his rival's strategy. Let us consider the following two cases:

- (a) Provider 2 does not react optimally to  $r_1(\cdot)$ . Then, there should be  $c_2 \in [\underline{c}_2, \bar{c}_2]$  such that  $r_2(c_2)$  does not maximize 2's expected utility. This implies that there is  $\hat{c}_1 < \bar{c}_1$  and  $\hat{x} \neq r_2(c_2)$  such that  $\hat{x}$  maximizes

$$\int_{\underline{c}_1}^{\hat{c}_1} \frac{\hat{b} - x}{2(\hat{b} - r_1(c_1))} (x - c_2) f_1(c_1) dc_1 + \int_{\hat{c}_1}^{\bar{c}_1} \left[ 1 - \frac{\hat{b} - r_1(c_1)}{2(\hat{b} - x)} \right] [x - c_2] f_1(c_1) dc_1.$$

Note that, since  $r_2(\cdot)$  is obtained by maximizing 2's expected utility conditional on  $r_2(c_2) \geq r_1(c_1)$  for each  $c_1$ , it must be the case that  $\hat{x} < r_2(c_2)$ . Maximizing the above expression we get the first order condition

$$\int_{\underline{c}_1}^{\hat{c}_1} \frac{c_2 + \hat{b} - 2\hat{x}}{2(\hat{b} - r_1(c_1))} f_1(c_1) dc_1 + \int_{\hat{c}_1}^{\bar{c}_1} \left[ 1 - \frac{(\hat{b} - r_1(c_1))(\hat{b} - c_2)}{2(\hat{b} - \hat{x})^2} \right] f_1(c_1) dc_1 = 0.$$



Since, as previously argued,  $\hat{x} < r_2(c_2)$ , we have that

$$\int_{\underline{c}_1}^{\hat{c}_1} \frac{c_2 + \hat{b} - 2\hat{x}}{2(\hat{b} - r_1(c_1))} f_1(c_1) dc_1 = \int_{\underline{c}_1}^{\hat{c}_1} \frac{r_2(c_2) - \hat{x}}{\hat{b} - r_1(c_1)} f_1(c_1) dc_1 > 0,$$

unless  $\underline{c}_1 = \hat{c}_1$ . This implies that either  $\underline{c}_1 \neq \hat{c}_1$ , and

$$\int_{\hat{c}_1}^{\bar{c}_1} \left[ 1 - \frac{(\hat{b} - r_1(c_1))(\hat{b} - c_2)}{2(\hat{b} - \hat{x})^2} \right] f_1(c_1) dc_1 < 0;$$

or  $\underline{c}_1 = \hat{c}_1$  and the above integral trivially becomes 0. Now, consider the integral above. Notice that for each  $c_1 \in [\hat{c}_1, \bar{c}_1]$ ,  $\hat{x} \leq r_1(c_1)$ . Moreover, since  $\hat{x} < r_2(c_2)$ , it follows that  $2(\hat{b} - \hat{x}) > \hat{b} - c_2$ . Therefore, for each  $c_1 \in [\hat{c}_1, \bar{c}_1]$ ,

$$\left[ 1 - \frac{(\hat{b} - r_1(c_1))(\hat{b} - c_2)}{2(\hat{b} - \hat{x})^2} \right] > 0,$$

which implies that the integral above is positive. A contradiction.

- (b) Provider 1 does not react optimally to  $r_2(\cdot)$ . Then, there should be  $c_1 \in [\underline{c}_1, \bar{c}_1]$  such that  $r_1(c_1)$  does not maximize 1's expected utility. This implies that there is  $\hat{c}_2 < \bar{c}_2$  and  $\hat{y} \neq r_1(c_1)$  such that  $\hat{y}$  maximizes

$$\int_{\underline{c}_2}^{\hat{c}_2} \frac{\hat{b} - y}{2(\hat{b} - r_2(c_2))} (y - c_1) f_2(c_2) dc_2 + \int_{\hat{c}_2}^{\bar{c}_2} \left[ 1 - \frac{\hat{b} - r_2(c_2)}{2(\hat{b} - y)} \right] [y - c_1] f_2(c_2) dc_2.$$

Note that the arguments above, concerning supplier 2, can be adapted here to find a contradiction.

Therefore, it is proven that strategies  $r_1(\cdot)$  and  $r_2(\cdot)$  constitute a Bayesian equilibrium for the contest stage subgame. More than that, our arguments are sufficient to guarantee that for any two continuous strategies  $r'_1(\cdot)$  and  $r'_2(\cdot)$  constituting a Bayesian equilibrium for the contest stage subgame, if  $r'_1(c_1) \leq r'_2(c_2)$  for any  $c_1 \in [\underline{c}_1, \bar{c}_1]$  and each  $c_2 \in [\underline{c}_2, \bar{c}_2]$ , it must be the case that for each provider  $i = 1, 2$ ,  $r'_i(\cdot) = r_i(\cdot)$ . ■

As pointed out in the above constructive proof, when Condition (C1) is satisfied, there is a natural Bayesian equilibrium for the contest stage. The less efficient provider believes that his revised price exceeds the one chosen by his rival. Therefore, provider 2 selects a revised price that depends only on his private information. Then, the most

efficient provider selects his strategy, under the assumption that his revised price is lower than the one of his rival. This strategy depends on the expected cost of his rival. Condition (C1) guarantees that the beliefs by the two providers, namely that  $r_1(c_1) \leq r_2(c_2)$ , are true for any  $c_1$  and each  $c_2$ . When, as illustrated by Example 2, Condition (C1) is not satisfied and the providers exhibit the “natural” beliefs about the ranking of revised prices, these beliefs might be not satisfied, contradicting the equilibrium conditions.

### A.3 Incomplete information and loser regret

This Appendix considers the two-stage process of Section 3 with incomplete information and loser regret. We assume that the price reports at the price discovery stage are made public and investigate the existence of a symmetric equilibrium in which suppliers report their marginal costs truthfully at the first stage, so that the contest stage is played under complete information.

Before formalizing a notion of regret, we investigate the behaviour of providers at the contest stage. Assume that  $P$  is the vector of prices selected at the price discovery stage, so that  $\hat{b} = \hat{t}(P) + \epsilon$  is the endogenous reserve price at the contest stage. Suppose that providers  $i$  and  $j$  participate at the contest stage.

Suppose that  $i$  believes that he knows the true value of  $c_j$ , and that he believes to be able to influence the beliefs of provider  $j$  by selecting  $p_i$  at the price discovery stage. In other words, at the contest stage  $j$  believes that  $c_i = p_i$ .

According to Alcalde and Dahm (2013), supplier  $j$ 's revised price at the contest stage is

$$r_j(p_i) = \begin{cases} \frac{c_j + \hat{b}}{2} & \text{if } c_j > p_i \\ \hat{b} - \sqrt{\frac{(\hat{b} - c_j)(\hat{b} - p_i)}{4}} & \text{if } c_j \leq p_i \end{cases}$$

Since  $i$  anticipates  $r_j(p_i)$  at the contest stage, at the price discovery stage  $i$  selects the price  $p_i$  that maximizes his profit, conditional on being one of the competitors at the contest stage. Figure 1 below represents two such functions for  $\hat{b}$  and  $c_j$  given. The dotted curve corresponds to the case in which  $c'_i < c_j$  holds, while the solid curve represents the situation with  $c''_i > c_j$ . The figure shows that, independently of whether  $i$  is more or less efficient than  $j$ , conditional on qualifying for the contest stage, at the price discovery stage it is optimal for provider  $i$  to inflate his signal about his costs.

Suppose that provider  $j$  updates his information about his rivals' costs according to a non-decreasing function  $f_j$ . More precisely, given  $p_i$  at the price discovery stage,  $j$

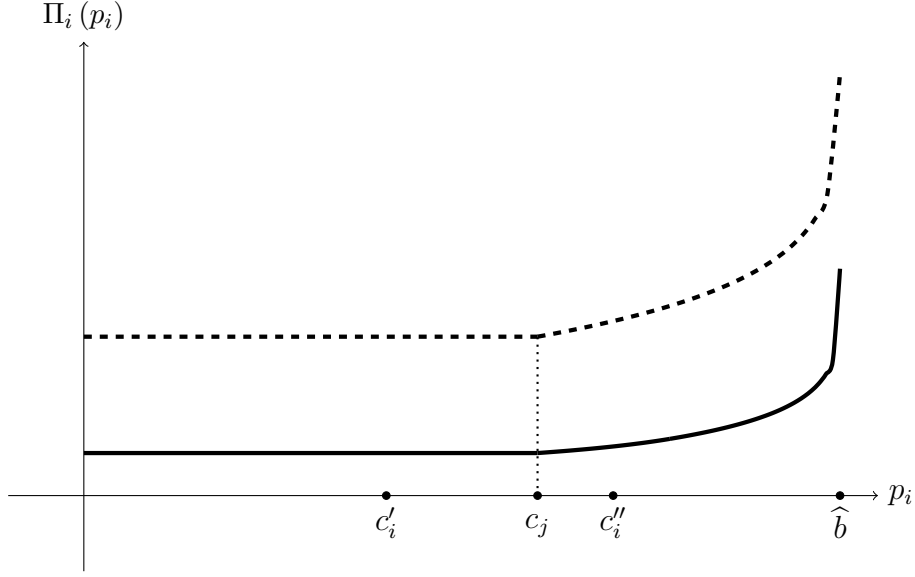


Figure 1: Supplier  $i$ 's profit

updates his beliefs about  $i$ 's costs to  $c_i^j = f_j(p_i)$ . Figure 2 below shows how  $j$ 's revised price at the contest stage depends on his beliefs about his rival's costs,  $c_i^j$ .

Note that when  $c_i^j < c_j$ , provider  $j$  believes that he is less efficient than his rival. Therefore, he expects his rival to select a lower revised price  $r_i^j$  at the contest stage, than he does and his best-reply is

$$\hat{r}_j = \frac{c_j + \hat{b}}{2}.$$

On the other hand, when  $c_i^j > c_j$ , provider  $j$  believes that he is more efficient than his rival. Anticipating that  $i$ 's costs are  $c_i^j$ , supplier  $j$  believes that  $i$ 's revised price at the contest stage is

$$\hat{r}_i^j = \frac{c_i^j + \hat{b}}{2}.$$

This implies that  $j$ 's best-reply is to select

$$r_j^*(\hat{r}_i^j) = \hat{b} - \sqrt{\frac{(\hat{b} - c_j)(\hat{b} - \hat{r}_i^j)}{2}}.$$

Figure 2 illustrates that provider  $j$ 's revised price selected at the contest stage increases with his belief about his rival's cost. Moreover, conditional on being a second stage contestant, since  $i$ 's share increases on  $r_j$  for any given values for  $r_i$  and  $\hat{b}$ , so does his profit.

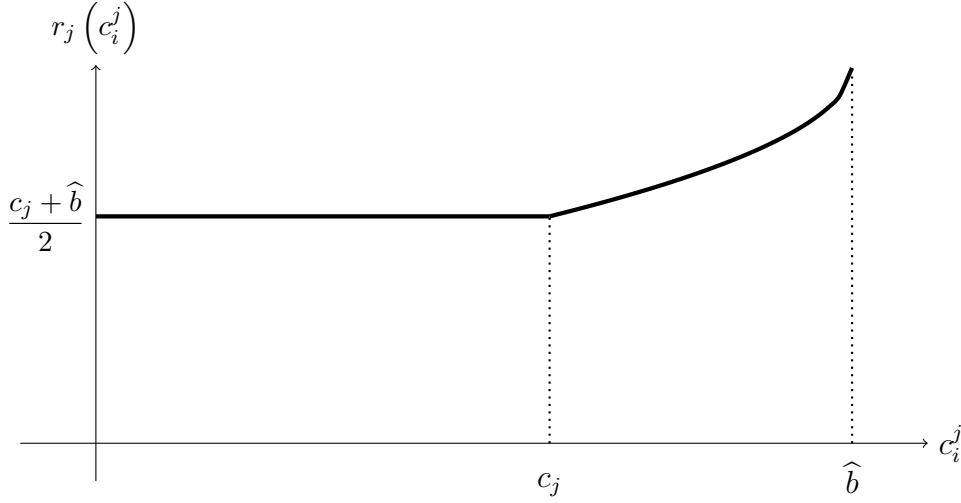


Figure 2: How  $r_j$  varies with  $c_i^j$

Thus, provider  $i$ 's price at the price discovery stage must resolve a trade-off. On one hand, inflating his price above cost might mislead his rival to be less competitive at the contest stage. But, on the other hand, inflating his price risks not to qualify for the contest stage. This latter concern is magnified by the introduction of loser regret.

One possible formalization of regret is the following variation of Filiz-Ozbay and Ozbay (2007). Consider a given supplier, say  $i$ , and a price report vector  $P_{-i} = (p_j)_{j \neq i}$  of his rivals, and let  $p_{(1)}$  and  $p_{(2)}$  be the lowest competing prices. More precisely, let  $p_{(1)} \leq p_{(2)} \leq p_j$  for each  $j \notin \{(1), (2), i\}$ . Consider the following utility of provider  $i$  of participating in the procurement auction

$$u_i(p_i | P_{-i}) = \begin{cases} -g(c_i - p_i) & \text{if } p_i \leq p_{(2)} < c_i \text{ and } i \in \mathcal{C}(P) \\ -g(p_i - c_i) & \text{if } c_i < p_{(2)} \leq p_i \text{ and } i \notin \mathcal{C}(P) \\ \Pi_i & \text{otherwise} \end{cases}, \quad (19)$$

where, for  $P$  given,  $\mathcal{C}(P)$  is the set of suppliers competing at the contest stage and  $g(\cdot)$  is a strictly increasing function satisfying that  $g(0) = 0$ . Notice that when  $i$  participates at the contest stage and  $p_i \leq p_{(2)} < c_i$  provider  $i$  experiences winner regret, while loser regret occurs when  $c_i < p_{(2)} \leq p_i$  and  $i$  is excluded from the contest stage.

Consider the price discovery stage and suppose that  $i$  chooses  $p_i \neq c_i$ , while his rivals select prices  $P_{-i}$ . There are two possibilities.

- (a)  $p_i < c_i$ . Supplier  $i$  either experiences winner regret and obtains a negative level of utility, or he has neither winner nor loser regret. In the latter case, the utility level of  $i$  associated to both  $p_i$  and  $p'_i = c_i$  is the same, and equal to his profit. Therefore,  $i$  has no incentive to select  $p_i < c_i$  at the price discovery stage.

- (b)  $p_i > c_i$ . As explained before, provider  $i$ 's price at the price discovery stage must resolve the trade-off of inducing his rival to be less competitive at the contest stage and managing the risk of not qualifying for the contest stage. When the loser regret is an important enough concern (i.e., when  $g(\cdot)$  is high enough for positive values), the incentive to misreport is counterbalanced.

This completes the argument that when loser regret is an important enough concern, then misreporting at the discovery stage is not profitable.

#### A.4 Proof of Proposition 1

We show that deviating from  $a^*$  at any  $\tilde{m} \in [0, 1)$  a provider can never increase his profits but in some circumstances may decrease it. Consider a provider, say  $i$ , and any  $\tilde{m} \in [0, 1)$ . Notice that if  $m_i < \tilde{m}$  holds, then  $i$ 's action at  $\tilde{m}$  does not affect his profits. Hence assume that  $i$ 's action at each  $m < \tilde{m}$  was  $a_{im} = 0$ . Consider the following cases, which exhaust all possibilities.

- (a)  $L(\tilde{m})$  has at least three providers. If  $a_{i\tilde{m}} = 1$ , then  $p_i = (1 - \tilde{m})b \geq \hat{t}(P)$ , which implies that  $s_i^{\text{EP}}(P) = 0$ . Notice that this is true even when there are other providers who also drop out in this moment. Suppose provider  $i$  compares dropping out at  $\tilde{m}$  to dropping out at  $m' > \tilde{m}$  and denote by  $m'' \geq \tilde{m}$  the next moment in which a provider  $j \neq i$  with  $j \in L(\tilde{m})$  drops out of the auction. There are two possibilities.
- (i)  $c_i \geq (1 - \tilde{m})b$ . Then  $m''$  must be such that  $c_i \geq (1 - m'')b$ . There are again two possibilities. If  $m' \leq m''$ , then provider  $i$ 's share and profits are zero. If  $m' > m''$ , then provider  $i$ 's share is strictly positive and his profits are strictly negative, because  $m' > \tilde{m}$  implies that  $c_i > (1 - m')b$ . Therefore, as prescribed by (11) and (12), it is optimal to select  $a_{i\tilde{m}} = 1$ , which guarantees zero profits.
  - (ii)  $c_i < (1 - \tilde{m})b$ . There are again two possibilities. If  $m''$  is such that  $c_i < (1 - m'')b$ , then provider  $i$  can choose  $m'$  as prescribed by (11) and (12) and obtain strictly positive profits. If  $m''$  is such that  $c_i \geq (1 - m'')b$ , then provider  $i$  can again follow (11) and (12) and choose  $m' = 1 - c_i/b$ , guaranteeing himself zero profits.

This implies that when  $L(\tilde{m})$  has at least three providers it is profitable to remain active until the price reaches marginal costs, as described in (11) and (12).

- (b)  $L(\tilde{m})$  has two providers. This implies that when the auction closes and all prices are determined,  $p_i \leq (1 - \tilde{m})b < \hat{t}(P)$ . Moreover, at moment  $\tilde{m}$  the value of  $\hat{t}(P)$

is publicly known. Therefore, by Theorem 4 in Alcalde and Dahm (2013) we have that  $i$ 's optimal decision is as described in (11) and (12).

- (c)  $i$  is the only provider in  $L(\tilde{m})$ . This implies that  $i$  is the supplier proposing the lowest price. Again, by Theorem 4 in Alcalde and Dahm (2013), the unique optimal decision for  $i$  is as described in (11) and (12).

This concludes the proof that  $a^*$  is a weakly dominant strategy.

## A.5 Multiple Sourcing

This Appendix investigates how the two-stage procedure in Section 3 can be extended to multiple sourcing. This is of interest, as dual sourcing is a special case of multiple sourcing and the latter is an important strategy to avoid the risk of lock-in with suppliers.<sup>33</sup>

To fix ideas suppose the buyer aims to assign shares to  $2 \leq \ell < n$  providers. The case of  $\ell = 2$  corresponds to the setting in Section 3. At the first stage each provider sets a price  $p_i$ , so that the vector  $P = (p_1, \dots, p_i, \dots, p_n)$  is determined. This allows to compute  $\hat{t}_\ell(P)$  the  $(\ell + 1)$ -th lowest price in  $P$ . To be precise,  $\hat{t}_\ell(P) \in \{p_1, \dots, p_i, \dots, p_n\}$  is such that at most  $\ell$  providers set a price below  $\hat{t}_\ell(P)$  and at least  $\ell + 1$  providers select a price that does not exceed this threshold. The game  $\Gamma^{\text{E}, \ell}$  is the variation of the two-stage game  $\Gamma^{\text{E}}$  in which  $\ell$  suppliers are assigned shares. That is, at the price discovery stage each provider selects a price  $p_i$ . This determines the endogenous reserve price  $\hat{t}_\ell(P)$ . Each of the  $\ell$  providers that compete at the contest stage is assigned a share as described in expression (1).

Although  $\Gamma^{\text{E}, \ell}$  is a simple generalization of  $\Gamma^{\text{E}}$ , it turns out that it might not preserve some of the properties of  $\Gamma^{\text{E}}$ . The following Example 3 sheds further light on these issues. In particular, there might be an equilibrium at which the ordering of prices differs from the ordering of costs.<sup>34</sup>

**Example 3** At the contest stage there are three suppliers with costs  $C = (50, 100, 103)$  and the fourth lowest price at the price discovery stage is 150. Simple computation identifies two Nash equilibria,  $\hat{R} = (\hat{r}_1, \hat{r}_2, \hat{r}_3)$  and  $\tilde{R} = (\tilde{r}_1, \tilde{r}_2, \tilde{r}_3)$ , which are described in the following table.

<sup>33</sup>Carpinetti et al. (2006) report that multiple awarding by European procurement agencies requires at least three suppliers. Engel et al. (2006) discuss advantages and drawbacks of multiple sourcing.

<sup>34</sup>In this example the “locally-envy-free” refinement of Edelman et al. (2007) is violated. This portends to the possibility of obtaining uniqueness of equilibrium at the contest stage by applying a refinement. Such an analysis, however, is beyond the scope of this paper and hence relegated to future research.

	Prices			Shares			Cost
	$r_1$	$r_2$	$r_3$	$S_1(R)$	$S_2(R)$	$S_3(R)$	$\sum r_i S_i(R)$
$\widehat{R}$	107.13	121.08	126.50	0.57	0.25	0.18	114.10
$\widetilde{R}$	107.57	125.00	122.33	0.58	0.20	0.23	114.36

Note that at equilibrium  $\widetilde{R}$  we have that  $\widetilde{r}_3 < \widetilde{r}_2$  holds, even though it is assumed that  $c_2 < c_3$ .

It follows from the arguments in the proof of Proposition 2 below that conditional on competing at the contest stage, a provider  $i$  can anticipate positive and negative profits in a similar way as in the case of  $\ell = 2$  (as formalised in Observation 1). Consequently, for each supplier  $i$  the truthful price report  $p_i = c_i$  still weakly dominates every other report at the price discovery stage.

The next result provides an equilibrium at the contest stage.

**Proposition 2** The contest stage of the game  $\Gamma^{\text{E},\ell}$  has a Nash Equilibrium  $R^{\text{O}}$  such that  $r_i^{\text{O}} \leq r_{i+1}^{\text{O}}$  for each provider  $i < \ell$ .

**Proof.** From the arguments above, it follows that providers reveal their costs at the price discovery stage, i.e. for each  $i$ ,  $p_i = c_i$ . Therefore, since  $0 \leq c_1 < c_2 < \dots < c_\ell < \widehat{t}_\ell(P)$ , providers 1 to  $\ell$  compete at the contest stage. For notational convenience we set  $r_{\ell+1}^{\text{O}} = \widehat{t}_\ell(P) = c_{\ell+1}$ .

Assume that revised prices  $r_i$  selected at the second stage are increasingly ordered (i.e.  $r_i \leq r_j$  whenever  $i < j$ ). Then, provider  $i$ 's share is

$$S_i(R) = \sum_{j=i}^{\ell} \frac{\min\{r_{\ell+1}^{\text{O}}, r_{j+1}\} - \min\{r_{\ell+1}^{\text{O}}, r_j\}}{j(r_{\ell+1}^{\text{O}} - r_1)}.$$

Consider the following procedure to obtain an ordered vector  $R^{\text{O}}$  of revised prices for the second stage contestants:

- (1) Supplier  $\ell$  solves the problem

$$\left. \begin{array}{l} \max_{r_\ell} (r_\ell - c_\ell) S_\ell(R) \\ \text{s. t. } \forall j < \ell, r_j = 0 \end{array} \right\}$$

which has the following unique solution

$$r_\ell^{\text{O}} = \frac{r_{\ell+1}^{\text{O}} + c_\ell}{2};$$

(2) each  $i, i < \ell$ , solves the problem

$$\left. \begin{array}{ll} \max_{r_i} & (r_i - c_i) S_i(R) \\ \text{s. t.} & \forall j < i, r_j = 0 \\ & \forall j > i, r_j = r_j^O \end{array} \right\} \quad (20)$$

which has the following unique solution for  $1 < i < n$ ,<sup>35</sup>

$$r_i^O = \frac{c_i}{2} + \frac{r_{i+1}^O}{2} + \frac{i}{2} \sum_{j=i+1}^{\ell} \frac{r_{j+1}^O - r_j^O}{j}.$$

Note that the above expressions can be also derived by solving the following  $\ell$  optimization problems

$$\max_{r_i} (r_i - c_i) S_i(R),$$

where for contestant  $i$ ,  $R = (r_i, R_{-i}^O)$  is constrained to satisfy (a)  $r_j = r_j^O$  for each  $j \neq i$  and (b)  $r_{i-1}^O \leq r_i \leq r_{i+1}^O$ .

We now show that  $R^O$  constitutes a Nash equilibrium for the contest stage subgame. Note that, otherwise, there should be a contestant  $i$  and revised price for him, say  $r'_i$  such that

$$(r'_i - c_i) S_i(r'_i, R_{-i}^O) > (r_i^O - c_i) S_i(R^O). \quad (21)$$

Note that the objective function for contestant  $i$ ,  $\Pi_i(r_i, R_{-i}^O) = (r_i - c_i) S_i(r_i, R_{-i}^O)$ , as a function of his revised price, satisfies the following properties:

- (a) It is positive for each  $r_i \in (c_i, r_{i+1}^O)$ . This is because  $r_i > c_i$  and  $S_i(r_i, R_{-i}^O)$  is also positive.
- (b) It is strictly increasing for each  $r_i \in (0, r_{i-1}^O)$ ; and
- (c) its partial derivative

$$\frac{\partial \Pi_i}{\partial r_i}(r_i, R_{-i}^O)$$

is continuous for each  $r_i \in (r_{i-1}^O, r_{i+1}^O)$ , where  $r_0^O = 0$ .

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<sup>35</sup>Unfortunately, for  $\ell \geq 3$ , there is no simple analytical expression for  $r_1^O$ . Nevertheless, since  $(r_1 - c_1) S_1(R)$  is strictly quasi-concave in  $r_1$ , program (20) has a unique solution for provider 1. Since  $c_1 < c_2$ , we have that  $r_1 < r_2$ .



Moreover, for  $\widehat{R}$  such that  $\widehat{r}_j = 0$  for  $j < i$  and  $\widehat{r}_j = r_j^{\mathbf{O}}$  for  $j > i$ , we have that for each  $r_i'' \in (r_{i-1}^{\mathbf{O}}, r_{i+1}^{\mathbf{O}})$

$$\text{sign} \frac{\partial \Pi_i}{\partial r_i} (r_i'', \widehat{R}_{-i}) = \text{sign} \frac{\partial \Pi_i}{\partial r_i} (r_i'', R_{-i}^{\mathbf{O}}).$$

Therefore, by program (20), it must be the case that  $r_i' > r_{i-1}^{\mathbf{O}}$ .

First, assume that  $i \neq 1$ . Then, if  $r_i' > r_{\ell}^{\mathbf{O}}$ ,

$$\frac{\partial \Pi_i}{\partial r_i} (r_i', R_{-i}^{\mathbf{O}}) = \frac{r_{\ell+1}^{\mathbf{O}} + c_i - 2r_i'}{\ell (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} < \frac{r_{\ell+1}^{\mathbf{O}} + c_i - 2r_{\ell}^{\mathbf{O}}}{\ell (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} = \frac{c_i - c_{\ell}}{\ell (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} < 0.$$

Then, since  $\Pi_i (r_i, R_{-i}^{\mathbf{O}})$  is decreasing for each  $r_i > r_{\ell}^{\mathbf{O}}$ , there is no loss of generality in assuming that  $r_i' \leq r_{\ell}^{\mathbf{O}}$ . When  $i = \ell - 1$ , since  $r_i^{\mathbf{O}}$  solves program (20), a contradiction is reached. Otherwise, suppose that  $r_{\ell-1}^{\mathbf{O}} < r_i' \leq r_{\ell}^{\mathbf{O}}$ . Therefore,

$$\begin{aligned} \frac{\partial \Pi_i}{\partial r_i} (r_i', R_{-i}^{\mathbf{O}}) &= \frac{r_{\ell+1}^{\mathbf{O}} - r_{\ell}^{\mathbf{O}}}{\ell (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} + \frac{r_{\ell}^{\mathbf{O}} + c_i - 2r_i'}{(\ell - 1) (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} < \\ &< \frac{r_{\ell+1}^{\mathbf{O}} - r_{\ell}^{\mathbf{O}}}{\ell (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} + \frac{r_{\ell}^{\mathbf{O}} + c_i - 2r_{\ell-1}^{\mathbf{O}}}{(\ell - 1) (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} < \\ &< \frac{c_i - c_{\ell-1}}{(\ell - 1) (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} < 0. \end{aligned}$$

As previously argued, since  $\Pi_i (r_i, R_{-i}^{\mathbf{O}})$  is decreasing for each  $r_i > r_{\ell-1}^{\mathbf{O}}$ , there is no loss of generality in assuming that  $r_i' \leq r_{\ell-1}^{\mathbf{O}}$ . When  $i = \ell - 2$ , provided that  $r_i^{\mathbf{O}}$  solves program (20), we obtain a contradiction. Otherwise, we can assume that  $r_{\ell-2}^{\mathbf{O}} < r_i' \leq r_{\ell-1}^{\mathbf{O}}$  and proceed as before. Note that an iterative argument yields that for any  $j > i$  and  $r_i'$  in  $(r_j^{\mathbf{O}}, r_{j+1}^{\mathbf{O}}]$ ,

$$\frac{\partial \Pi_i}{\partial r_i} (r_i', R_{-i}^{\mathbf{O}}) < \frac{c_i - c_j}{j (r_{\ell+1}^{\mathbf{O}} - r_1^{\mathbf{O}})} < 0,$$

which implies that  $r_{i-1}^{\mathbf{O}} \leq r_i' \leq r_{i+1}^{\mathbf{O}}$ , and thus  $r_i' = r_i^{\mathbf{O}}$ . A contradiction.

Now, let assume that  $i = 1$ . In such a case the above arguments can be replicated, just by taking into account that for  $j > 1$  and  $r_1' \in (r_j^{\mathbf{O}}, r_{j+1}^{\mathbf{O}}]$ ,

$$\frac{\partial \Pi_1}{\partial r_1} (r_1', R_{-1}^{\mathbf{O}}) < \frac{c_1 - c_j}{j (r_{\ell+1}^{\mathbf{O}} - r_2^{\mathbf{O}})} < 0,$$

which is sufficient to reach a contradiction. ■

We conclude with an illustrative example.

**Example 4** There are  $n = 7$  (potential) providers. The costs are summarized in  $C = (38, 65, 71, 74, 80, 100, 145)$ . The initial reserve price is  $b = 196$ . The buyer wants to select the  $\ell = 5$  most efficient suppliers. Then, as explained before at the price discovery stage each supplier sets price  $p_i$  equal to his cost. Therefore, the revised reserve price at the second stage is  $\hat{t}_\ell(P) = c_6 = 100$ .

To compute the equilibrium described in Proposition 2 we proceed as follows. First, provider 5 computes the price that maximizes his profits when it is the maximal price selected by any of the contestants. This price is

$$r_5^O = \frac{\hat{t}_\ell(P) + c_5}{2} = \frac{100 - 80}{2} = 90.$$

Then, provider 4, anticipating that  $r_5^O$  is 90, computes the price that maximizes his profit, taking into account that it should not exceed  $r_5^O$  and that the remaining 3 providers select lower prices. This price is

$$r_4^O = \frac{r_5^O + c_4}{2} + \frac{4}{2} \frac{\hat{t}_\ell(P) - r_5^O}{5} = \frac{90 + 74}{2} + \frac{4}{2} \frac{100 - 90}{5} = 86,$$

and so on. In this way we obtain the vector  $R^O = (69, 78, 83, 86, 90)$ .

To verify that this constitutes a Nash equilibrium for the contest stage we observe that the continuous profit function  $\Pi_i(r_i, R_{-i}^O)$  of provider  $i$ , given the prices selected by his rivals,

- (a) is increasing for values  $r_i < r_{i-1}^O$ , and
- (b) is decreasing for values  $r_i > r_{i+1}^O$ .

Therefore, the best response for provider  $i$  to his rivals' messages  $R_{-i}^O$  follows from maximizing  $\Pi_i(r_i, R_{-i}^O)$  for  $r_{i-1}^O \leq r_i \leq r_{i+1}^O$ . This best response is precisely  $r_i^O$ .

In Figure 3 we plot how the profit of each provider varies as a function of his revised prices, given his rivals' revised prices  $R_{-i}^O$ . To be precise, for supplier  $i$  given, we consider the function  $\Pi_i^O(r_i) = \Pi_i(r_i, R_{-i}^O)$ . As mentioned in the proof of Proposition 2, for each supplier  $i$  participating in the contest stage,

- (a)  $\Pi_i^O$  is continuous in  $r_i$ ;
- (b)  $\Pi_i^O$  is increasing at each  $r_i < r_i^O$ ;
- (c)  $\Pi_i^O$  is decreasing at each  $r_i > r_i^O$ ; and
- (d)  $\Pi_i^O$  is differentiable almost everywhere. To be precise, if there is  $r_i$  at which  $\Pi_i^O$  is not differentiable, then there is another provider  $j \neq i$  such that  $r_i = r_j^O$ .



Figure 3: Supplier  $i$ 's profit